MA30118 - Question Sheet Five

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If you wish to have this sheet marked, please place it in the wallet on my office door, 1W4.8. I will mark it and place it in the pigeon-holes in the Maths square for you to collect and notify you via e-mail when it is there. Obviously, for this to work, you must ensure your full name is on the work!

1. The values of Bath building contracts (in millions of pounds) for a 12-month period are (in time order) 17, 21, 19, 23, 18, 16, 20, 18, 22, 20, 15, 22

   (a) Use $\alpha = 0.1$ to compute the exponential smoothing values for the time series using the month 1 figure as the initial forecast. Calculate the forecast error for each forecast.

   (b) What is the forecast for next month? And the month following that?

   (c) What is the value of the mean square deviation (MSD)?

2. Let X_t denote the quarterly earning per share of a corporation in quarter t. Using a season of four, a Holt-Winters multiplicative model with smoothing parameters $\alpha = 0.5$, $\gamma = 0.6$ and $\delta = 0.7$ was fitted. The data is summarised below.

t	X_t	M_t	T_t	S_t
1	0.712	0.5287729	0.01678876	1.3616536
2	0.584	0.6020379	0.05067450	0.9450805
3	0.620	0.6900502	0.07307715	0.8846493
4	0.620	0.7430797	0.06104857	0.8413058
5	0.891	0.7292399	0.01611553	1.2637702
6	0.570	0.6742393	-0.02655412	0.8753022
7	0.540			
8	0.690			

   (a) Find M_7, T_7 and S_7.

   (b) Find M_8, T_8 and S_8.

   (c) Find the forecast of the quarterly earning per share in quarters 9, 10 and 15.

3. Let β_0 and β_1 be constants, $s_t = s_{t-12}$ for all t, and $\{I_t\}$ a sequence of independent random quantities with zero mean and constant variance.

   (a) If $X_t = (\beta_0 + \beta_1 t)s_t + I_t$, show that $\nabla_{12}^2 X_t$ is a stationary process.

   (b) If $X_t = \beta_0 + \beta_1 t + s_t + I_t$, show that $\nabla\nabla_{12} X_t$ is a stationary process.

4. Forecasting from a fitted Box-Jenkins model. As mentioned in class such forecasts are obtained as follows. Suppose we have fitted the $ARMA(2,1)$ model:

$$X_s \;=\; \phi_1 X_{s-1} + \phi_2 X_{s-2} + a_s + \theta_1 a_{s-1},$$

and that the latest available value is X_t. We wish to forecast ahead at $s = t{+}1, t{+}2, \ldots$. Write down the forecasts $\hat{X}_{t+1}$, $\hat{X}_{t+2}$ and $\hat{X}_{t+3}$ using the following rules:

   (a) Use the above equation with the estimates $\hat{\phi}_1$, $\hat{\phi}_2$ and $\hat{\theta}_1$ and

   (b) if the equation involves a_s where $s \leq t$ use the residual from the fit, $\hat{a}_s = X_s - \hat{X}_s$, otherwise if $s > t$, use $a_s = 0$, its expected value,

   (c) if the equation involves X_s where $s \leq t$ use the actual recorded value of X_t but if $s > t$ use the forecast $\hat{X}_s$.

5. The $ARIMA(1,1,1)$ model, which has first-order differencing, can be written in the form

$$W_t \;=\; \gamma + \phi_1 W_{t-1} + a_t - \theta_1 a_{t-1}$$

where $W_t = \nabla X_t$.

   (a) Express the model in terms of $\{X_t\}$ and $\{a_t\}$ rather than $\{W_t\}$ and $\{a_t\}$.

   (b) Use this to derive the forecasts for one, two and three steps ahead from time t using the usual rules.