MA30118 - Question Sheet Two

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Attempt all questions. Hand in by 17:00 Friday 10th March either to me in lectures, or the envelope on my door, 1W4.8.

1. For a fee of £400, the manager in question 2. of Question Sheet One may call in a consultant. Let I_1 represent the event that the consultant predicts that business will decline, I_2 that the consultant predicts business will remain the same, I_3 that the consultant predicts business will increase moderately, and I_4 that the consultant predicts business will increase rapidly. Table 1 lists the conditional probabilities for predictions made by the consultant.

| | S_1 | S_2 | S_3 | S_4 |
|-------|-------|----------------|-------|-------|
| I_1 | 0.80 | 0.10 | 0.20 | 0.10 |
| I_2 | 0.10 | $0.10 \\ 0.70$ | 0.20 | 0.20 |
| I_3 | 0.05 | 0.10 | 0.50 | 0.30 |
| I_4 | 0.05 | 0.10 | 0.10 | 0.40 |

Table 1: Conditional probabilities for the consultant's predictions.

- (a) Find the EVSI.
- (b) What is the net efficiency?
- (c) Comment on the value of the consultant.
- 2. I have been offered two investment opportunities, A and B, which require approximately the same cash outlay. The cash requirements mean that I can only afford to make at most one investment. I thus have three alternatives: make investment $A(A_1)$; make investment $B(A_2)$; or to not invest (A_3) . The returns on my investments depends upon what happens to the stock market in the next year. With probability 0.3, the stock market may increase (S_1) ; with probability 0.5 it may remain stable (S_2) and with probability 0.2, the market may fall (S_3) . The possible payoffs, in pounds as profits, are given in Table 2 below.
 - (a) Calculate the EMV of the investment decision problem and thus state the optimal decision under this criterion.
 - (b) Suppose that it is pointed out to me that both actions A_1 and A_2 could result in losses, so I decide to think about the risk of the investments. I elect to construct

| | S_1 | S_2 | S_3 |
|-------|--------|---------|---------|
| A_1 | 45,000 | 30,000 | -75,000 |
| A_2 | 75,000 | -30,000 | -45,000 |
| A_3 | 0 | 0 | 0 |

Table 2: Payoffs for my investment choices in question 2.

a utility function over my possible payoffs. I assign the following indifference probabilities.

| Profit | Indifference probability |
|------------------|--------------------------|
| $\pounds 45,000$ | 0.95 |
| $\pm 30,000$ | 0.90 |
| $\pounds 0$ | 0.75 |
| $-\pounds30,000$ | 0.55 |
| -£45,000 | 0.40 |

Construct the corresponding utility table and hence find the decision which maximises the expected utility. Comment on this decision.

3. A firm has three investment alternatives: A_1 , A_2 , and A_3 . The return of these investments depends upon what happens to the stock market in the next year. With probability 0.4, the stock market may go up (S_1) ; with probability 0.3 it may remain stable (S_2) and with probability 0.3, the market may go down (S_3) . The possible payoffs, in \$1000s as profits, are given in Table 3 below.

| | S_1 | S_2 | S_3 |
|-------|-------|-------|-------|
| A_1 | 100 | 25 | 0 |
| A_2 | 75 | 50 | 25 |
| A_3 | 50 | 50 | 50 |

Table 3: Payoffs for my investment choices in question 3.

- (a) Calculate the EMV of the investment decision problem and thus state the optimal decision under this criterion.
- (b) For the lottery having a payoff of \$100,000 with probability p and \$0 with probability 1-p, two decision makers expressed the following indifference probabilities:

| | Indifference probability | | |
|-----------|--------------------------|------------------|--|
| Profit | Decision Maker A | Decision Maker B | |
| \$75,000 | 0.80 | 0.60 | |
| \$ 50,000 | 0.60 | 0.30 | |
| \$ 25,000 | 0.30 | 0.15 | |

Find the most preferred decision for each decision maker using the expected utility approach.

(c) Why don't decision makers A and B select the same decision alternative?