MA30118 - Question Sheet One

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Attempt all questions. Hand in by 17:00 Tuesday 21st February either to me in lectures, or the envelope on my door, 1W4.8.

1. Lincoln City are considering the number of programmes to print for Saturday's big match. They believe that there is a 35% chance that there will be a heavy turnout (S_1), a 50% chance for a normal turnout (S_2), and a 15% chance for a low turnout (S_3). They must decide whether to print 500 copies (A_1), 750 copies (A_2), 1000 copies (A_3), or 1250 copies (A_4). The payoff table, Table 1, is given below. Unsold programmes would result in a loss.

	S_1	S_2	S_3
A_1	100	100	100
A_2	150	140	110
A_3	200	160	75
A_4	250	120	-50

Table 1: Payoffs for programme production

   (a) Are all the actions admissible?

   (b) Find the minimax regret decision.

   (c) Find the decision which maximises expected payoff.

2. A manager is considering his staffing needs and has the following strategies: Lay off two staff employees (A_1); maintain staff at current levels (A_2); increase the staff size by one employee (A_3); increase the staff size by two employees (A_4). There are four possible states of nature: Business will decrease (S_1); business will stay the same (S_2); business will increase moderately (S_3); and business will increase rapidly (S_4). The prior probabilities are $P(S_1) = 0.15$, $P(S_2) = 0.25$, $P(S_3) = 0.25$ and $P(S_4) = 0.35$. The possible payoffs, in pounds as profits, are given in Table 2 overleaf. Find the decision which maximises expected payoff.

3. A firm decides to build a new factory which will last 10 years. Let h_1 denote the event that the demand is high for the first two years, l_1 the event that the demand is low for the first two years, h_2 that the demand is high for the last eight years and l_2 that the demand is low for the last eight years. These are the only possible outcomes.

	S_1	S_2	S_3	S_4
A_1	1451	1840	2050	2300
A_2	-1091	1685	2430	2900
A_3	-2015	1100	3060	3561
A_4	-3460	-1350	3340	4300

Table 2: Payoffs for the choice of staffing levels.

Marketing information reveals that

$$\begin{aligned} P(h_1, h_2) &= P(h_1 \cap h_2) &= 0.6, \\ P(h_1, l_2) &= P(h_1 \cap l_2) &= 0.1, \\ P(l_1, l_2) &= P(l_1 \cap l_2) &= 0.3. \end{aligned}$$

Find $P(h_1)$, $P(h_2|h_1)$ and $P(l_2|h_1)$.

4. The firm in question 3. may either build a big factory or a small factory which may then be expanded. If it builds a big factory and demand is high for the full ten years then the profit is £14million. If it builds big and demand is high for the first two years and low for the remaining eight then the profit is $-$£0.4million. A big factory with low demand for the entire ten years gives a profit of $-$£4million. If the firm builds a small factory and demand is high for the first two years then it may expand the factory. Expansion followed by eight years of high demand yields a profit of £6million, while expansion followed by eight years of low demand results in a profit of -£4.4million. If the factory is not expanded and demand remains high, then the profit is £3.2million, while if the demand falls to low for the remaining eight years then the profit is £5.6million. A small plant with low demand for the whole ten years returns a profit of £5.4million.

   (a) Construct the decision tree for the firm.

   (b) Find the expected value of perfect information (EVPI) for the firm.

   (c) Which decision maximises the expected payoff. Comment on this choice of action.