

**MA30118: MANAGEMENT STATISTICS**  
**Assessed Coursework: Quality Control**

**Preamble**

**Set:** Monday 24 April 2006.

**Due in:** Please hand the work in to the Mathematical Sciences Department Office, 1W3.12 by 12 noon on Friday 12 May 2006.

**Notes on submission:** The work should be accompanied by a completed Coursework Cover Sheet, available either in lectures or from the Departmental Office. Your submitted assignment should be stapled in the top left hand corner and must **not** be presented in either a folder or a plastic wallet. Your answers may be handwritten, typed or a mixture of both. You may use any computer package you like to construct any of the required charts but all working should be clearly shown.

**Time:** The average student should spend no more than ten hours on this assignment.

**Conditions:** The assignment should be your own work, though you are free to discuss it with your peers. You may also consult me for general advice. It should be completed in your own time.

**Value:** This assignment carries 100% of the total marks for coursework, and 40% of the total marks for the course.

**Contact details**

Simon Shaw

Room: 1W4.8

E-mail: [s.c.shaw@maths.bath.ac.uk](mailto:s.c.shaw@maths.bath.ac.uk)

1. A production process produces primer paint. The viscosity of the paint is used as a measure of its quality and you have been employed to set up  $\bar{x}$  and  $R$ -charts to monitor the process. In order to set up the charts, 25 samples, each of size five, have been taken when the process is thought to be in control. The viscosity data from these samples are shown in Table 1.

| $j$ | $x_{j1}$ | $x_{j2}$ | $x_{j3}$ | $x_{j4}$ | $x_{j5}$ | $\bar{x}_j$ | $r_j$  |
|-----|----------|----------|----------|----------|----------|-------------|--------|
| 1   | 34.1968  | 34.2000  | 34.1840  | 34.2112  | 34.1920  | 34.19680    | 0.0272 |
| 2   | 34.2000  | 34.1744  | 34.2080  | 34.1968  | 34.1936  | 34.19456    | 0.0336 |
| 3   | 34.2032  | 34.1936  | 34.1888  | 34.2240  | 34.2144  | 34.20480    | 0.0352 |
| 4   | 34.1744  | 34.2032  | 34.2048  | 34.2080  | 34.1952  | 34.19712    | 0.0336 |
| 5   | 34.2096  | 34.1472  | 34.1904  | 34.2000  | 34.1744  | 34.18432    | 0.0624 |
| 6   | 34.2240  | 34.2128  | 34.1888  | 34.2000  | 34.2160  | 34.20832    | 0.0352 |
| 7   | 34.2480  | 34.2032  | 34.2304  | 34.1872  | 34.2128  | 34.21632    | 0.0608 |
| 8   | 34.1920  | 34.2096  | 34.1904  | 34.2000  | 34.2080  | 34.20000    | 0.0192 |
| 9   | 34.2064  | 34.1984  | 34.1840  | 34.2096  | 34.2144  | 34.20256    | 0.0304 |
| 10  | 34.2064  | 34.2000  | 34.2112  | 34.2000  | 34.1936  | 34.20224    | 0.0176 |
| 11  | 34.2128  | 34.1920  | 34.2144  | 34.2080  | 34.2064  | 34.20672    | 0.0224 |
| 12  | 34.1808  | 34.2384  | 34.2336  | 34.2080  | 34.2032  | 34.21280    | 0.0576 |
| 13  | 34.2144  | 34.1904  | 34.1952  | 34.1760  | 34.1888  | 34.19296    | 0.0384 |
| 14  | 34.2192  | 34.2224  | 34.1968  | 34.1984  | 34.2112  | 34.20960    | 0.0256 |
| 15  | 34.1920  | 34.1872  | 34.2016  | 34.2176  | 34.2064  | 34.20096    | 0.0304 |
| 16  | 34.1904  | 34.2192  | 34.1776  | 34.2080  | 34.2112  | 34.20128    | 0.0416 |
| 17  | 34.2160  | 34.1824  | 34.1840  | 34.2144  | 34.2224  | 34.20384    | 0.0400 |
| 18  | 34.1872  | 34.2112  | 34.2240  | 34.1824  | 34.2224  | 34.20544    | 0.0416 |
| 19  | 34.1904  | 34.1968  | 34.1904  | 34.1920  | 34.1840  | 34.19072    | 0.0128 |
| 20  | 34.2000  | 34.2160  | 34.2208  | 34.2320  | 34.2048  | 34.21472    | 0.0320 |
| 21  | 34.1760  | 34.2048  | 34.1888  | 34.2240  | 34.1808  |             |        |
| 22  | 34.2096  | 34.2160  | 34.2288  | 34.2048  | 34.2000  |             |        |
| 23  | 34.1728  | 34.2032  | 34.1968  | 34.1952  | 34.2192  |             |        |
| 24  | 34.1808  | 34.2016  | 34.2144  | 34.2080  | 34.1936  |             |        |
| 25  | 34.1712  | 34.1744  | 34.1920  | 34.2272  | 34.2208  |             |        |

Table 1: Viscosity of primer paint.

- (a) For samples  $j = 21$  to  $25$ , calculate the corresponding values of  $\bar{x}_j$  and  $r_j$ . Show your working. [5]
- (b) Using the data in Table 1, set up the  $R$ -chart with 3-sigma control limits. Does the process variability appear to be in control? [8]
- (c) Now construct the  $\bar{x}$ -chart, again with 3-sigma control limits. Is there any indication that the process was out of control when the data was collected? [8]

After the control charts were set up, 15 additional samples, each of size five, from the process were collected. The data from these new samples are shown in Table 2.

- (d) Do the  $\bar{x}$  and  $R$ -charts you have set up suggest that the process is in control for these additional samples? Explain your findings. [10]
- (e) It is argued that the addition of upper and lower warning limits, placed two standard deviations away from the centre line, on each chart will improve the

| $j$ | $x_{j1}$ | $x_{j2}$ | $x_{j3}$ | $x_{j4}$ | $x_{j5}$ | $\bar{x}_j$ | $r_j$  |
|-----|----------|----------|----------|----------|----------|-------------|--------|
| 26  | 34.2192  | 34.2240  | 34.2480  | 34.1776  | 34.2000  | 34.21376    | 0.0704 |
| 27  | 34.1920  | 34.2160  | 34.1840  | 34.2240  | 34.2016  | 34.20352    | 0.0400 |
| 28  | 34.1792  | 34.1984  | 34.1760  | 34.2000  | 34.1840  | 34.18752    | 0.0240 |
| 29  | 34.2128  | 34.2160  | 34.2048  | 34.1856  | 34.2096  | 34.20576    | 0.0304 |
| 30  | 34.2048  | 34.2000  | 34.2016  | 34.1776  | 34.1952  | 34.19584    | 0.0272 |
| 31  | 34.1904  | 34.2048  | 34.2240  | 34.2320  | 34.2064  | 34.21152    | 0.0416 |
| 32  | 34.2128  | 34.2032  | 34.2288  | 34.1920  | 34.2080  | 34.20896    | 0.0368 |
| 33  | 34.2016  | 34.2064  | 34.1840  | 34.1936  | 34.1968  | 34.19648    | 0.0224 |
| 34  | 34.2240  | 34.2000  | 34.2256  | 34.2400  | 34.2000  | 34.21792    | 0.0400 |
| 35  | 34.2480  | 34.2080  | 34.2000  | 34.2256  | 34.2192  | 34.22016    | 0.0480 |
| 36  | 34.2016  | 34.1840  | 34.1920  | 34.2160  | 34.2384  | 34.20640    | 0.0544 |
| 37  | 34.2240  | 34.2320  | 34.2384  | 34.2080  | 34.2304  | 34.22656    | 0.0304 |
| 38  | 34.2560  | 34.2160  | 34.2192  | 34.2240  | 34.2416  | 34.23136    | 0.0400 |
| 39  | 34.2272  | 34.2208  | 34.2576  | 34.2400  | 34.2416  | 34.23744    | 0.0368 |
| 40  | 34.2160  | 34.2240  | 34.2464  | 34.2000  | 34.2320  | 34.22368    | 0.0464 |

Table 2: Viscosity of primer paint, samples taken after the control charts have been set up.

sensitivity of the charts. How do the addition of these limits affect the conclusions you made in part (d)? [5]

- (f) Suppose that the company has cut the budget available to monitor the process. In future, at each time point, a sample of size four rather than five will be taken. As a result of this, explain carefully any changes you would make to the control charts you have set up. [10]

2. You are a management trainee in a manufacturing company that uses control charts, with mixed success, to control several different production lines. Your boss has heard about the following article:

Alwan, L.C. and Roberts, H.V. (1988) Time-Series Modeling for Statistical Process Control. *Journal of Business & Economic Statistics*, **6(1)**, 87-95.

The article highlights some limitations of the traditional implementation of statistical process control and the assumptions underpinning the theory. The authors propose an approach to process control using time-series models. Your boss, knowing you had studied process control during your degree course, asks you to read the paper and summarise its findings.

Write a report, in your own words, summarising what you consider to be the main points made by Alwan and Roberts and explaining the principles involved. You may assume that your boss has some, but not an extensive, knowledge of statistics. Your boss is a busy person, so you should aim to be as concise as possible, and rambling answers should be avoided. Moreover, you have not been asked to write a general account of process control. Your boss does not expect that you will know every technical term or concept, which is used in the paper, but expects that you can meet the challenge of producing a good report even though there may be a few things in the paper you are not familiar with. Your report should be no more than 2000 words long (and may indeed be shorter). [54]