Previously in MA30087/50087:

- fundamental theorem of linear programming: if there is an optimal solution, there is an optimal basic feasible solution
  - let $\mathbf{x}$ be an optimal feasible solution
  - wlog, let non-zero components of $\mathbf{x}$ be first k components
  - if the corresponding columns of $\mathbf{A}$, $\mathbf{a}_1, \ldots, \mathbf{a}_k$ are linearly independent then we're done: $\mathbf{x}$ is a basic feasible solution
  - otherwise, assume $\mathbf{a}_1, \ldots, \mathbf{a}_k$ are linearly dependent so that there exist constants α_j not all equal to 0 such that

$$\sum_{j=1}^{k} \alpha_j \mathbf{a}_j = \mathbf{0}_m$$

  - we'll show that as long as the $\mathbf{a}_1, \ldots, \mathbf{a}_k$ are linearly dependent we can find an optimal solution with one fewer zero value than $\mathbf{x}$

Today in MA30087/50087:

- complete proof of fundamental theorem
- Simplex method: search for optimal basic feasible solution by moving from extreme point to adjacent extreme point in a direction that improves the objective function
- $\mathbf{A} = (\mathbf{B} \,|\, \mathbf{N})$, $\mathbf{x} = \begin{pmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} \mathbf{c}_B \\ \mathbf{c}_N \end{pmatrix}$ then

$$\mathbf{x}_B + \mathbf{B}^{-1}\mathbf{N}\mathbf{x}_N = \mathbf{B}^{-1}\mathbf{b}$$

$$z = \mathbf{c}^T\mathbf{x} = \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{b} + \left(\mathbf{c}_N^T - \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{N}\right)\mathbf{x}_N$$

- these tell us all that we need to know
- $\mathbf{r}^T = \mathbf{c}_N^T - \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{N}$ represent the reduced costs