Previously in MA30087/50087:

- consider solutions to LP problem in canonical form. Feasible set

$$F_{(C)} \;=\; \{\mathbf{x} \in \mathbb{R}^n \,:\, \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}_n\}$$

- basis: choose m linearly independent columns, corresponding variables are basic variables
  - wlog, assume these are first m columns and set $\mathbf{B} = (\mathbf{a}_1, \ldots, \mathbf{a}_m)$ and $\mathbf{N} = (\mathbf{a}_{m+1}, \ldots, \mathbf{a}_n)$ so that $\mathbf{A} = (\mathbf{B} \,|\, \mathbf{N})$
  - $\mathbf{B}$ is an invertible $m \times m$ matrix
- basic solution: $\mathbf{x}$ solves $\mathbf{A}\mathbf{x} = \mathbf{b}$ with non-basic variables equal to zero
  - let $\mathbf{x}_B = (x_1, \ldots, x_m)^T$ and $\mathbf{x}_N = (x_{m+1}, \ldots, x_n)^T$ then

$$\mathbf{x}_B + \mathbf{B}^{-1}\mathbf{N}\mathbf{x}_N \;=\; \mathbf{B}^{-1}\mathbf{b}$$

  - $\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}$, $\mathbf{x}_N = \mathbf{0}_{n-m}$ solves $\mathbf{A}\mathbf{x} = \mathbf{b}$
- basic feasible solution: basic solution with $\mathbf{x} \geq \mathbf{0}_n$
- extreme point: no distinct $\mathbf{v}$ and $\mathbf{w}$ such that, for some $\lambda \in (0, 1)$,

$$\mathbf{x} \;=\; \lambda\mathbf{v} + (1 - \lambda)\mathbf{w}$$

- if $\mathbf{x} \in F_{(C)}$ is a basic feasible solution then $\mathbf{x}$ is an extreme point

Today in MA30087/50087:

- if $\mathbf{x} \in F_{(C)}$ is an extreme point then it is a basic feasible solution
- fundamental theorem of linear programming: if there is an optimal solution, there is an optimal basic feasible solution
- remember: if $\mathbf{a}_1, \ldots, \mathbf{a}_k$ are linearly dependent then there exist $\alpha_j \in \mathbb{R}$ not all equal to zero such that

$$\sum_{j=1}^{k} \alpha_j \mathbf{a}_k \;=\; \mathbf{0}_m$$