Previously in MA30087/50087:

- relating LP problem to convex sets
  - objective function is a hyperplane which is a (closed) convex set
  - constraints of LP problem form a closed convex polyhedron
- solve LP-problem in canonical form, that is find $\mathbf{x} \in \mathbb{R}^n$ that will

$$\begin{aligned} \text{maximise} \quad & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n \end{aligned}$$

  where $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{b} \in \mathbb{R}^m$, $\mathbf{c} \in \mathbb{R}^n$ and $\mathbf{0}_n$ is the zero vector
- feasible set is

$$F_{(C)} \;=\; \{\mathbf{x} \in \mathbb{R}^n \,:\, \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}_n\}$$

Today in MA30087/50087:

- assume that $m \leq n$ with $\mathrm{rank}(\mathbf{A}) = m$
- $\mathbf{A} = (\mathbf{a}_1, \ldots, \mathbf{a}_n)$ so that

$$\mathbf{A}\mathbf{x} \;=\; \sum_{j=1}^{n} x_j \mathbf{a}_j$$

- basis: choose m linearly independent columns, corresponding variables are basic variables
  - wlog, assume these are first m columns and set $\mathbf{B} = (\mathbf{a}_1, \ldots, \mathbf{a}_m)$ and $\mathbf{N} = (\mathbf{a}_{m+1}, \ldots, \mathbf{a}_n)$ so that $\mathbf{A} = (\mathbf{B}\,|\,\mathbf{N})$
  - $\mathbf{B}$ is an invertible $m \times m$ matrix
- basic solution: $\mathbf{x}$ solves $\mathbf{A}\mathbf{x} = \mathbf{b}$ with non-basic variables equal to zero
  - let $\mathbf{x}_B = (x_1, \ldots, x_m)^T$ and $\mathbf{x}_N = (x_{m+1}, \ldots, x_n)^T$ then

$$\mathbf{x}_B + \mathbf{B}^{-1}\mathbf{N}\mathbf{x}_N \;=\; \mathbf{B}^{-1}\mathbf{b}$$

  - $\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}$, $\mathbf{x}_N = \mathbf{0}_{n-m}$ solves $\mathbf{A}\mathbf{x} = \mathbf{b}$
- basic feasible solution: basic solution with $\mathbf{x} \geq \mathbf{0}_n$