Previously in MA30087/50087:

- balanced transportation problem

$$\begin{array}{lrl} \text{minimise} & z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} & \\ \text{subject to} & \sum_{j=1}^{n} x_{ij} = s_i & i = 1, \ldots, m \\ & \sum_{i=1}^{m} x_{ij} = d_j & j = 1, \ldots, n \\ & x_{ij} \geq 0 & i = 1, \ldots, m, j = 1, \ldots, n \end{array}$$

- dual of the balanced transportation problem

$$\begin{array}{lrl} \text{maximise} & z' = \sum_{i=1}^{m} u_i s_i + \sum_{j=1}^{n} v_j d_j & \\ \text{subject to} & u_i + v_j \leq c_{ij} & i = 1, \ldots, m, j = 1, \ldots, n \end{array}$$

- transportation algorithm utilises asymmetric complementary slackness

$$x_{ij}[u_i + v_j - c_{ij}] \; = \; 0 \; \forall \; i = 1, \ldots, m, j = 1, \ldots n$$

- find an initial BFS to balanced transportation problem
  - typically use north-west corner method or the matrix method
- solve complementary slackness to find possible dual solution

$$x_{ij} > 0 \;\Rightarrow\; u_i + v_j = c_{ij}$$

- <u>if dual solution feasible, we have optimal solution</u>
- if dual solution not feasible, move to another BFS

Today in MA30087/50087:

- using the matrix method to find an initial BFS
- coping with degeneracy: keep $m + n - 1$ variables in the basis
- for $x_{ij} = 0$ in the basis, still solve $u_i + v_j = c_{ij}$