Previously in MA30087/50087:

- balanced transportation problem

$$\begin{aligned} \text{minimise} \quad & z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\ \text{subject to} \quad & \sum_{j=1}^{n} x_{ij} = s_i \quad i = 1, \ldots, m \\ & \sum_{i=1}^{m} x_{ij} = d_j \quad j = 1, \ldots, n \\ & x_{ij} \geq 0 \quad i = 1, \ldots, m, j = 1, \ldots, n \end{aligned}$$

- dual of the balanced transportation problem

$$\begin{aligned} \text{maximise} \quad & z' = \sum_{i=1}^{m} u_i s_i + \sum_{j=1}^{n} v_j d_j \\ \text{subject to} \quad & u_i + v_j \leq c_{ij} \quad i = 1, \ldots, m, j = 1, \ldots, n \end{aligned}$$

- transportation algorithm utilises asymmetric complementary slackness

$$x_{ij}[u_i + v_j - c_{ij}] = 0 \ \forall \ i = 1, \ldots, m, j = 1, \ldots n$$

Today in MA30087/50087:

- find an initial BFS to balanced transportation problem

  – typically use north-west corner method or the matrix method

- solve complementary slackness to find possible dual solution

$$x_{ij} > 0 \ \Rightarrow \ u_i + v_j = c_{ij}$$

- if dual solution feasible, we have optimal solution

- if dual solution not feasible, move to another BFS

- example of the transportation algorithm

- perturbation loop for moving to another BFS