Previously in MA30087/50087:

- transporting goods from sources $S_1, \ldots, S_m$ with supplies $s_1, \ldots, s_m$ to destinations $D_1, \ldots, D_n$ with demands $d_1, \ldots, d_n$
- amount transported from S_i to D_j is x_{ij}
- cost of transporting a unit from S_i to D_j is c_{ij}
- balanced transportation problem (so $\sum_{i=1}^m s_i = \sum_{j=1}^n d_j$)

$$\begin{array}{lrl} \text{minimise} & z = & \displaystyle\sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{subject to} & & \displaystyle\sum_{j=1}^n x_{ij} = s_i \quad i = 1, \ldots, m \\ & & \displaystyle\sum_{i=1}^m x_{ij} = d_j \quad j = 1, \ldots, n \\ & & x_{ij} \geq 0 \quad i = 1, \ldots, m, j = 1, \ldots, n \end{array}$$

Today in MA30087/50087:

$$\mathbf{Ax} = \mathbf{b} \quad \Rightarrow \quad \begin{pmatrix} 1 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 & \cdots & 1 \\ 1 & \cdots & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & \cdots & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} x_{11} \\ \vdots \\ x_{1n} \\ \vdots \\ x_{m1} \\ \vdots \\ x_{mn} \end{pmatrix} = \begin{pmatrix} s_1 \\ \vdots \\ s_m \\ d_1 \\ \vdots \\ d_n \end{pmatrix}$$

- dual of the balanced transportation problem, $\mathbf{y} = (u_1, \ldots, u_m, v_1, \ldots, v_n)^T$

$$\begin{array}{lrl} \text{maximise} & z' = & \displaystyle\sum_{i=1}^m u_i s_i + \sum_{j=1}^n v_j d_j \\ \text{subject to} & & u_i + v_j \leq c_{ij} \quad i = 1, \ldots, m, j = 1, \ldots, n \end{array}$$

- balanced transportation problem has a finite optimal solution
- $\text{Rank}(\mathbf{A}) \; = \; m + n - 1$
- transportation algorithm utilises asymmetric complementary slackness

$$x_{ij}[u_i + v_j - c_{ij}] \; = \; 0 \; \forall \; i = 1, \ldots, m, j = 1, \ldots n$$