Previously in MA30087/50087:

- aymmetric complementary slackness theorem

$$(P) \quad \begin{array}{ll} \text{maximise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n. \end{array}$$

- $\mathbf{x} \in \mathbb{R}^n$ is an optimal feasible solution for (P) and $\mathbf{y} \in \mathbb{R}^m$ is an optimal feasible solution for the corresponding dual (D) if and only if they are feasible solutions and

$$x_j[(\mathbf{A}^T\mathbf{y})_j - c_j] \; = \; 0 \; \forall j = 1, \ldots, n$$

which is equivalent to

$$x_j > 0 \;\Rightarrow\; (\mathbf{A}^T\mathbf{y})_j = c_j \;\; \mathit{or} \;\; (\mathbf{A}^T\mathbf{y})_j > c_j \;\Rightarrow\; x_j = 0.$$

Today in MA30087/50087:

- transporting goods from sources $S_1, \ldots, S_m$ with supplies $s_1, \ldots, s_m$ to destinations $D_1, \ldots, D_n$ with demands $d_1, \ldots, d_n$
- amount transported from S_i to D_j is x_{ij}
- cost of transporting a unit from S_i to D_j is c_{ij}
- the transportation problem is to

$$\begin{array}{lrl} \text{minimise} & z = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij} & \\ \text{subject to} & \sum_{j=1}^n x_{ij} \leq s_i & i = 1, \ldots, m \\ & \sum_{i=1}^m x_{ij} \geq d_j & j = 1, \ldots, n \\ & x_{ij} \geq 0 & i = 1, \ldots, m, j = 1, \ldots, n \end{array}$$

- problem is balanced if $\sum_{i=1}^m s_i = \sum_{j=1}^n d_j$