Previously in MA30087/50087:

- primal (P) and symmetric dual (D) for the standard maximisiation problem

$$
\text{(P)} \quad \begin{array}{rr} \text{maximise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n \end{array} \qquad \text{(D)} \quad \begin{array}{rr} \text{minimise} & z' = \mathbf{b}^T\mathbf{y} \\ \text{subject to} & \mathbf{A}^T\mathbf{y} \geq \mathbf{c} \\ & \mathbf{y} \geq \mathbf{0}_m \end{array}
$$

- Corollary 3
  - if there exists at least one feasible solution to both (P) and (D) then there exist bounded optimal solutions to (P) and (D) with equal objective functions
- (Symmetric) Complementary Slackness Theorem

  Consider the standard LP problem (P). Then $\mathbf{x}$, $\mathbf{y}$ optimal feasible solutions for (P) and (D) respectively if and only if they are feasible solutions and

$$
\begin{aligned} y_i[(\mathbf{A}\mathbf{x})_i - b_i] &= 0 \ \forall i = 1, \ldots, m, \\ x_j[(\mathbf{A}^T\mathbf{y})_j - c_j] &= 0 \ \forall j = 1, \ldots, n. \end{aligned}
$$

Today in MA30087/50087:

- Asymmetric Complementary Slackness Theorem

  Consider the canonical LP problem. Then $\mathbf{x}$, $\mathbf{y}$ optimal feasible solutions for the canonical problem and its dual respectively if and only if they are feasible solutions and

$$
x_j[(\mathbf{A}^T\mathbf{y})_j - c_j] = 0 \ \forall j = 1, \ldots, n
$$

  which is equivalent to

$$
x_j > 0 \ \Rightarrow \ (\mathbf{A}^T\mathbf{y})_j = c_j \ \ \textit{or} \ \ (\mathbf{A}^T\mathbf{y})_j > c_j \ \Rightarrow \ x_j = 0.
$$