Previously in MA30087/50087:

- primal (P) and symmetric dual (D) for the standard maximisiation problem

$$\text{(P)}\quad \begin{array}{rr} \text{maximise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n \end{array} \qquad \text{(D)}\quad \begin{array}{rr} \text{minimise} & z' = \mathbf{b}^T\mathbf{y} \\ \text{subject to} & \mathbf{A}^T\mathbf{y} \geq \mathbf{c} \\ & \mathbf{y} \geq \mathbf{0}_m \end{array}$$

- weak duality theorem: $\mathbf{x}_0$ is a feasible solution to (P) and $\mathbf{y}_0$ is a feasible solution to its dual (D) then $\mathbf{c}^T\mathbf{x}_0 \leq \mathbf{b}^T\mathbf{y}_0$

Today in MA30087/50087:

- Corollary 1
  - (P) feasible but unbounded $\Rightarrow$ (D) not feasible
  - (D) feasible but unbounded $\Rightarrow$ (P) not feasible
- Corollary 2
  - $\mathbf{x}_0$ feasible for (P), $\mathbf{y}_0$ feasible for (D) with $\mathbf{c}^T\mathbf{x}_0 = \mathbf{b}^T\mathbf{y}_0 \Rightarrow \mathbf{x}_0$ and $\mathbf{y}_0$ optimal
- Duality Theorem: (P) has a finite optimal solution $\Leftrightarrow$ (D) has a finite optimal solution in which case optimal value of the objective functions are the same
- Corollary 3
  - if there exists at least one feasible solution to both (P) and (D) then there exist bounded optimal solutions to (P) and (D) with equal objective functions