

Previously in MA30087/50087:

- **duality**: motivating example

$$\begin{array}{ll}\text{maximise} & z = 1x_1 + 2x_2 \\ \text{subject to} & 5x_1 + 20x_2 \leq 400 \\ & 10x_1 + 15x_2 \leq 450 \\ & x_1, x_2 \geq 0.\end{array}$$

- maximum is $z = 52$ obtained when $x_1 = 24$ and $x_2 = 14$

Today in MA30087/50087:

- consider linear combinations of the **constraints**, multipliers $y_1, y_2 \geq 0$ such that

$$\begin{array}{ll} 5y_1 + 10y_2 \geq 1 & (x_1 \text{ value in objective}) \\ 20y_1 + 15y_2 \geq 2 & (x_2 \text{ value in objective}) \end{array}$$

which make the upper bound, $400y_1 + 450y_2$, as **small** as possible

- **primal problem** (standard maximisation problem)

$$\begin{array}{ll}\text{maximise} & z = \mathbf{c}^T \mathbf{x} \\ \text{(P) subject to} & \mathbf{Ax} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n\end{array}$$

- **symmetric dual problem** (is in standard minimisation form)

$$\begin{array}{ll}\text{minimise} & z' = \mathbf{b}^T \mathbf{y} \\ \text{(D) subject to} & \mathbf{A}^T \mathbf{y} \geq \mathbf{c} \\ & \mathbf{y} \geq \mathbf{0}_m\end{array}$$

- dual of the dual is the primal
- **asymmetric dual of the canonical maximisation problem**: **no constraints** on the sign of the variables