**Previously in MA30087/50087:**

- Two-phase simplex method
  - in the first phase we solve an auxiliary LP problem to obtain a basic feasible solution to the problem we want to solve

$$\begin{array}{ll} \text{maximise} & z' = -\sum_{j=1}^{k} u_j \\ \text{subject to} & \mathbf{A}'\mathbf{x}' = \mathbf{b} \\ & \mathbf{x}' \geq \mathbf{0}_{n+k} \end{array}$$

  - in the second phase we use this basic feasible solution as the starting point to solve the original LP problem (with $\mathbf{b} \geq \mathbf{0}_m$)

$$\begin{array}{ll} \text{maximise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n \end{array}$$

**Today in MA30087/50087:**

- the primal simplex algorithm
- duality: motivating example

$$\begin{array}{lrcl} \text{maximise} & z & = & 1x_1 + 2x_2 \\ \text{subject to} & 5x_1 + 20x_2 & \leq & 400 \\ & 10x_1 + 15x_2 & \leq & 450 \\ & x_1, x_2 & \geq & 0. \end{array}$$

- maximum is $z = 52$ obtained when $x_1 = 24$ and $x_2 = 14$
- consider linear combinations of the constraints, multipliers $y_1, y_2 \geq 0$ such that

$$\begin{array}{rcll} 5y_1 + 10y_2 & \geq & 1 & (x_1 \text{ value in objective}) \\ 20y_1 + 15y_2 & \geq & 2 & (x_2 \text{ value in objective}) \end{array}$$

  which make the upper bound, $400y_1 + 450y_2$, as small as possible