Previously in MA30087/50087:

- used the simplex algorithm with immediate initial bfs
- essentially solved $\mathbf{A}\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \geq \mathbf{0}_m$ and $\mathbf{A}$ contains $\mathbf{I}_m$ amongst its columns
- what do we do when we can't find an initial basic feasible solution?
- add further artificial variables and use a two-phase method

Today in MA30087/50087:

- in the first phase we solve an auxiliary LP problem to obtain a basic feasible solution to the problem we want to solve

$$\begin{array}{ll} \text{maximise} & z' = -\sum_{j=1}^{k} u_j \\ \text{subject to} & \mathbf{A}'\mathbf{x}' = \mathbf{b} \\ & \mathbf{x}' \geq \mathbf{0}_{n+k} \end{array}$$

- wlog, $\mathbf{A}'\mathbf{x}' = \mathbf{A}\mathbf{x} + \sum_{j=1}^{k} u_k \mathbf{e}_j$ (essentially we are adding sufficient artificial variables to constraints to ensure $\mathbf{A}'$ contains $\mathbf{I}_m$ amongst its columns)
- solution with $z' = 0$ corresponds to a basic feasible solution of the original problem
- in the second phase we use this basic feasible solution as the starting point to solve the original LP problem (with $\mathbf{b} \geq \mathbf{0}_m$)

$$\begin{array}{ll} \text{maximise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n \end{array}$$

- example of two-phase method