Previously in MA30087/50087:

- used the simplex algorithm with the slack variables as an initial basic feasible solution
- essentially solved $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{b} \geq \mathbf{0}_m$ and $\mathbf{A}$ contains $\mathbf{I}_m$ amongst its columns

Today in MA30087/50087:

- what do we do when we can't find an initial basic feasible solution?
- add further artificial variables and use a two-phase method
  1. Constraint of the form $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$ where $b_i \geq 0$
     * Add in a slack variable $s_i \geq 0$ so that $\sum_{j=1}^{n} a_{ij}x_j + s_i = b_i$
  2. Constraint of the form $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$ where $b_i < 0$
     * Add in a slack variable $s_i \geq 0$ so that $\sum_{j=1}^{n} a_{ij}x_j + s_i = b_i$
     * Multiply through by -1 and add in an artificial variable $u_i \geq 0$ so that $-\sum_{j=1}^{n} a_{ij}x_j - s_i + u_i = -b_i$
  3. Constraint of the form $\sum_{j=1}^{n} a_{ij}x_j = b_i$ where $b_i \geq 0$
     * Add in an artificial variable $u_i \geq 0$ so that $\sum_{j=1}^{n} a_{ij}x_j + u_i = b_i$
- in the first phase we solve an auxiliary LP problem to obtain a basic feasible solution to the problem we want to solve

$$\begin{array}{ll} \text{maximise} & z' = -\sum_{j=1}^{k} u_j \\ \text{subject to} & \mathbf{A}'\mathbf{x}' = \mathbf{b} \\ & \mathbf{x}' \geq \mathbf{0} \end{array}$$

- solution with $z' = 0$ corresponds to a basic feasible solution of the original problem