MA30087/50087 - Question Sheet Nine

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2015/16 Semester I

Set: Lecture/Problems Class, Monday 7th December 2015.
Due in: There is no deadline for this work. I will happily mark and return any work you hand in either to me or to my office, 4W4.10. Solution Sheets will be available in the revision lecture on Thursday 10th December2015.
Task: Attempt questions 1-3; questions 4-5 are additional questions which may be used for tutorial discussion.

1. Sources 1, 2, 3 stock peppermint rock in amounts of 20, 42, 18 tonnes respectively while demand at seaside resorts Cleethorpes (1), Skegness (2) and Mablethorpe (3) is 39, 34, 7 tonnes respectively. The matrix of transport costs is

$$\begin{pmatrix} 7 & 4 & 9 \\ 8 & 12 & 5 \\ 3 & 11 & 7 \end{pmatrix}$$

   where the (i, j)th entry corresponds to the route from source i to resort j. Find, using the north-west corner method for an initial basic feasible solution, the minimal cost transportation scheme.

2. Consider the transportation problem with costs $\mathbf{c}$, supply $\mathbf{s}$ and demand $\mathbf{d}$ given by

$$\mathbf{c} = \begin{pmatrix} 4 & 3 & 3 & 1 \\ 3 & 2 & 4 & 8 \\ 5 & 4 & 6 & 3 \end{pmatrix}, \mathbf{s} = \begin{pmatrix} 8 \\ 11 \\ 16 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} 4 \\ 9 \\ 9 \\ 13 \end{pmatrix}.$$

   Solve the problem, finding an initial configuration using the matrix method.

3. Consider the transportation problem with costs $\mathbf{c}$, supply $\mathbf{s}$ and demand $\mathbf{d}$ given by

$$\mathbf{c} = \begin{pmatrix} 6 & 8 & 5 \\ 7 & 4 & 6 \\ 9 & 5 & 6 \\ 2 & 4 & 3 \end{pmatrix}, \mathbf{s} = \begin{pmatrix} 30 \\ 40 \\ 50 \\ 10 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} 40 \\ 60 \\ 30 \end{pmatrix}.$$

   Solve the problem, finding an initial configuration using the north-west corner method.

4. The balanced transportation problem in matrix form is

$$\begin{array}{ll} \text{minimise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_{mn} \end{array}$$

where $\mathbf{A} = (\mathbf{e}_{11} \cdots \mathbf{e}_{1n} \cdots \mathbf{e}_{m1} \cdots \mathbf{e}_{mn})$, with $\mathbf{e}_{ij} = \begin{pmatrix} \mathbf{e}_i^{(m)} \\ \mathbf{e}_j^{(n)} \end{pmatrix} \in \mathbb{R}^{m+n}$ where $\mathbf{e}_i^{(m)}$ denotes the ith column of the $m \times m$ identity matrix and $\mathbf{e}_j^{(n)}$ the jth column of the $n \times n$ identity matrix, $\mathbf{b} = (s_1, \ldots, s_m, d_1, \ldots, d_n)^T$, $\mathbf{c} = (c_{11}, \ldots, c_{1n}, \ldots, c_{m1}, \ldots, c_{mn})^T$ and $\mathbf{x} = (x_{11}, \ldots, x_{1n}, \ldots, x_{m1}, \ldots, x_{mn})^T$. Prove that $\text{Rank}(\mathbf{A}) = m + n - 1$.

5. It is required to move machines from factories A, B and C to warehouses X, Y and Z. There are 5 required at X, 4 at Y and 3 at Z, whilst there are 8 available at A, 5 at B and 3 at C. The transport cost (in £) between the sites is as follows.

	X	Y	Z
A	50	60	30
B	60	40	20
C	40	70	30

Plan the movements to minimise total transport cost.