MA30087/50087 - Question Sheet Eight

Simon Shaw, s.shaw@bath.ac.uk
http://people.bath.ac.uk/masss/ma30087.html

2015/16 Semester I

Set: Problems Class, Thursday 26th November 2015.
Due in: Problems Class, Thursday 3rd December 2015. If you are unable to make the problems class then you should ensure that you hand the work to me personally *before* the problems class is held; my office is 4W4.10.
Task: Attempt questions 1-3; questions 4-5 are additional questions which may be used for tutorial discussion.

1. Consider the following linear programming problem

$$\begin{array}{ll} \text{maximise} & z = 2x_1 + 4x_2 + 3x_3 + x_4 \\ \text{subject to} & 3x_1 + x_2 + x_3 + 4x_4 \leq 12 \\ & x_1 - 3x_2 + 2x_3 + 3x_4 \leq 7 \\ & 2x_1 + x_2 + 3x_3 - x_4 \leq 10 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{array}$$

   Use the complementary slackness conditions to deduce whether or not $x_1 = 0$, $x_2 = \frac{52}{5}$, $x_3 = 0$, $x_4 = \frac{2}{5}$ is an optimal solution to the problem.

2. Consider the following linear programming problem

$$\begin{array}{ll} \text{maximise} & z = -2x_1 - 3x_2 - 4x_3 \\ \text{subject to} & x_1 + 2x_2 + x_3 = 3 \\ & 2x_1 - x_2 + 3x_3 = 4 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

   Use the complementary slackness conditions to deduce whether or not $x_1 = \frac{11}{5}$, $x_2 = \frac{2}{5}$, $x_3 = 0$ is an optimal solution to the problem.

3. Suppose that we consider the asymmetric dual to the primal canonical linear programming problem

$$(P) \quad \begin{array}{ll} \text{maximise} & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}_n \end{array}$$

   where $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$ and $\mathbf{c} \in \mathbb{R}^n$ are given. Prove that the conclusion of the Weak Duality Theorem is still valid. That is to say, for any feasible solution $\mathbf{x}$ to the primal (P) and feasible solution $\mathbf{y}$ to the corresponding dual (D) it is still true that $\mathbf{c}^T\mathbf{x} \leq \mathbf{b}^T\mathbf{y}$. In addition, show that it is still true that if $\mathbf{c}^T\mathbf{x} = \mathbf{b}^T\mathbf{y}$ then $\mathbf{x}$ and $\mathbf{y}$ are optimal for (P) and (D) respectively.

4. By considering how one may write the canonical linear programming problem as a standard maximisation problem, use the Duality Theorem for the symmetric dual to prove that the same result is true for the asymmetric dual.

5. Consider the following linear programming problem

$$
\begin{array}{ll}
\text{minimise} & z = x_1 + 6x_2 + 2x_3 - x_4 + x_5 - 3x_6 \\
\text{subject to} & x_1 + 2x_2 + x_3 + 5x_6 = 3 \\
 & -3x_2 + 2x_3 + x_4 + x_6 = 1 \\
 & 5x_2 + 3x_3 + x_5 - 2x_6 = 2 \\
 & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.
\end{array}
$$

Use the complementary slackness conditions to deduce whether or not $x_1 = 0$, $x_2 = \frac{16}{29}$, $x_3 = 0$, $x_4 = \frac{66}{29}$, $x_5 = 0$, $x_6 = \frac{11}{29}$ is an optimal solution to the problem.