MA30087/50087 - Question Sheet Six

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2015/16 Semester I

Set: Problems Class, Thursday 12th November 2015.
Due in: Problems Class, Thursday 19th November 2015. If you are unable to make the problems class then you should ensure that you hand the work to me personally *before* the problems class is held; my office is 4W4.10.
Task: Attempt questions 1-3; questions 4-5 are additional questions which may be used for tutorial discussion.

1. Solve the following linear programming problem by the two-phase method.

$$\begin{array}{lr} \text{maximise} & z = x_1 + x_2 - 2x_3 + 2x_4 \\ \text{subject to} & x_1 - x_2 - x_3 - 2x_4 \geq 2 \\ & x_1 + x_2 + x_4 \leq 8 \\ & x_1 + 2x_2 - x_3 = 4 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{array}$$

2. Solve the following linear programming problem.

$$\begin{array}{lr} \text{maximise} & z = 4x_1 + x_2 + 3x_3 + 5x_4 \\ \text{subject to} & -4x_1 + 6x_2 + 5x_3 - 4x_4 \leq 20 \\ & 3x_1 - 2x_2 + 4x_3 + x_4 \leq 10 \\ & 8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{array}$$

3. Consider the following linear programming problem.

$$\begin{array}{lr} \text{maximise} & z = 3x_1 - 9x_2 - 2x_3 - 4x_4 + x_5 + x_6 \\ \text{subject to} & x_1 - 8x_4 + 10x_6 = 1 \\ & x_2 + x_3 - 11x_4 + 13x_6 = 7 \\ & 5x_2 + \alpha x_4 + x_5 + 2x_6 = 12 \\ & x_1, \ldots, x_6 \geq 0 \end{array}$$

   where $\alpha \in \mathbb{R}$.

   (a) With the help of a simplex tableau, explain in detail why the objective, z, can be made arbitrarily large when $\alpha < -2$.
   [Hint: If you can immediately identify an initial basic feasible solution do so. Remember that your initial tableau should be a representation of the linear programming problem in the form $\mathbf{x}_B + \mathbf{B}^{-1}\mathbf{N}\mathbf{x}_N = \mathbf{B}^{-1}\mathbf{b}$, $z - (\mathbf{c}_N^T - \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{N})\mathbf{x}_N = \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{b}$ where $\mathbf{x}_B$ represent your initial basic variables and $\mathbf{x}_N$ your initial non-basic variables.]

(b) Show that when $\alpha > -2$ there is an optimal solution stating clearly the associated values of $x_1, \ldots, x_6$ and the optimal value of the objective.

(c) What happens when $\alpha = -2$?
[Hint: Think about the reduced costs and any corresponding consequence for the objective function.]

4. Using the two-phase simplex method, show that the following problem has the optimal solution $(x_1, x_2, x_3) = (1, 0, 10)$.

$$\begin{array}{lr} \text{maximise} & z = 2x_1 + x_2 + 3x_3 \\ \text{subject to} & x_1 - 3x_2 + \frac{1}{2}x_3 = 6 \\ & -x_1 + 8x_2 + \frac{1}{2}x_3 \leq 4 \\ & -3x_1 + 4x_2 + \frac{1}{2}x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

5. A student has been asked to solve a linear programming problem. After two steps of the two-phase method, the student has the following tableau

T	z'	z	x_1	x_2	x_3	s_1	s_2	u_1	u_2	u_3	u_4	
x_1	0	0	1	0	2	2	0	–	0	0	–	3
s_2	0	0	0	-5	-1	3	1	–	0	0	–	5
u_2	0	0	0	-3	5	1	0	–	1	0	–	5
u_3	0	0	0	-2	-3	0	0	–	0	1	–	1
II	0	1	0	1	-4	3	0	–	0	0	–	5
I	1	0	0	-5	-2	-1	0	–	0	0	–	-6

where x_1, x_2 and x_3 are the original variables, s_1 and s_2 are slack variables, and u_1, u_2, u_3 and u_4 are artificial variables, so that the first phase of the algorithm attempts to maximise $z' = -u_1 - u_2 - u_3 - u_4$. Explain briefly how it can be seen that the student must have made a mistake in implementing the algorithm.