MA30087/50087 - Question Sheet Four

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2015/16 Semester I

Set: Problems Class, Thursday 29th October 2015.
Due in: Problems Class, Thursday 5th November 2015. If you are unable to make the problems class then you should ensure that you hand the work to me personally *before* the problems class is held; my office is 4W4.10.
Task: Attempt questions 1-3; questions 4-5 are additional questions which may be used for tutorial discussion. In this homework, when using the simplex method, you are free to introduce a variable into the basis in any sensible fashion. I recommend using a basis consisting of the slack variables as an initial basic feasible solution.[1]

1. Recall the example that we discussed in Lectures to motivate the simplex method.

$$\begin{array}{lrcl} \text{maximise} & z & = & x_1 + 2x_2 \\ \text{subject to} & 5x_1 + 20x_2 + s_1 & = & 400 \\ & 10x_1 + 15x_2 + s_2 & = & 450 \\ & x_1, x_2, s_1, s_2 & \geq & 0. \end{array} \tag{1}$$

   In lectures, we started with the initial basis $\{s_1, s_2\}$ and then noted that the addition of either x_1 or x_2 to the basis would improve the objective function. We decided to introduce x_2 and proceeded to solve the problem using the simplex method. Now solve the problem by introducing x_1 to the initial basis $\{s_1, s_2\}$ and explain the steps of the simplex algorithm in terms of the feasible region of the problem (1) in its original standard form.

2. Consider the following linear programming problem.

$$\begin{array}{lr} \text{maximise} & z = x_1 + 3x_2 + 2x_3 \\ \text{subject to} & 3x_1 + x_2 + 2x_3 \leq 700 \\ & 2x_1 + 4x_2 \leq 1200 \\ & 4x_1 + 3x_2 + 8x_3 \leq 1000 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

   Use the simplex method to find the optimal solution.

3. A mining company produces 100 tons of red ore and 80 tons of black ore each week. These can be treated in different ways to produce three different alloys, Soft, Hard or Strong. To produce 1 ton of Soft alloy requires 5 tons of red ore and 3 tons of black. For the Hard alloy the requirements are 3 tons of red and 5 tons of black, whilst for the

[1] In these problems, the basis consisting of the slack variables always give a feasible solution as the origin lies in the feasible set of the problems in standard form.

Strong alloy they are 5 tons of red and 5 tons of black. The profit per ton from selling the alloys (after allowing for production but not mining costs, which are regarded as fixed) are £250, £300 and £400 for Soft, Hard and Strong respectively. Formulate the problem of deciding how much of each alloy to make each week as a linear programming problem and use the simplex method to find the optimal solution.

4. Solve, using the simplex method, the following linear programming problem

$$\begin{aligned} \text{maximise} \quad & z = 2x_1 - 3x_2 + x_3 \\ \text{subject to} \quad & 3x_1 + 6x_2 + x_3 \leq 6 \\ & 4x_1 + 2x_2 + x_3 \leq 4 \\ & x_1 - x_2 + x_3 \leq 3 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

5. Use the simplex method to show that the following problem has no finite maximising solution.

$$\begin{aligned} \text{maximise} \quad & z = -x_1 + 2x_2 + x_3 \\ \text{subject to} \quad & 3x_1 + x_2 - 4x_3 \leq 4 \\ & x_1 - x_2 - x_3 \leq 10 \\ & x_1 - 2x_2 + 6x_3 \leq 9 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Find a particular solution with $z > 1000$.