MA30087/50087 - Question Sheet Two

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2015/16 Semester I

Set: Problems Class, Thursday 15th October 2015.
Due in: Problems Class, Thursday 22nd October 2015. If you are unable to make the problems class then you should ensure that you hand the work to me personally *before* the problems class is held; my office is 4W4.10.
Task: Attempt questions 1-3; question 4 is an additional question which may be used for tutorial discussion.

1. Suppose the canonical form of a linear programming problem is given by the constraint matrix $\mathbf{A}$ and vector $\mathbf{b}$, where:

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 4 & 0 & 3 & 0 & 1 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 5 \\ 3 \\ 6 \end{pmatrix}.$$

   Determine which of the following points is

   (a) a feasible solution to the linear programming problem;

   (b) a basic solution;

   (c) a basic feasible solution.

      i. $\mathbf{x} = (0, 3, 0, 5, 6)^T$
      ii. $\mathbf{x} = (0, 3, 5, 0, -9)^T$
      iii. $\mathbf{x} = \left(\frac{3}{2}, 0, 0, \frac{1}{2}, 0\right)^T$
      iv. $\mathbf{x} = \left(\frac{1}{2}, 1, 1, 0, 2\right)^T$
      v. $\mathbf{x} = \left(1, 1, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}\right)^T$

2. Recall our motivating linear programming problem from Lecture 1.

$$\begin{array}{lrcl} \text{maximise} & z & = & x_1 + 2x_2 \\ \text{subject to} & 5x_1 + 20x_2 & \leq & 400 \\ & 10x_1 + 15x_2 & \leq & 450 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   (a) Write the problem in canonical form.

   (b) Determine all of the basic solutions of the problem in canonical form and interpret them in terms of the constraints of the problem in standard form.

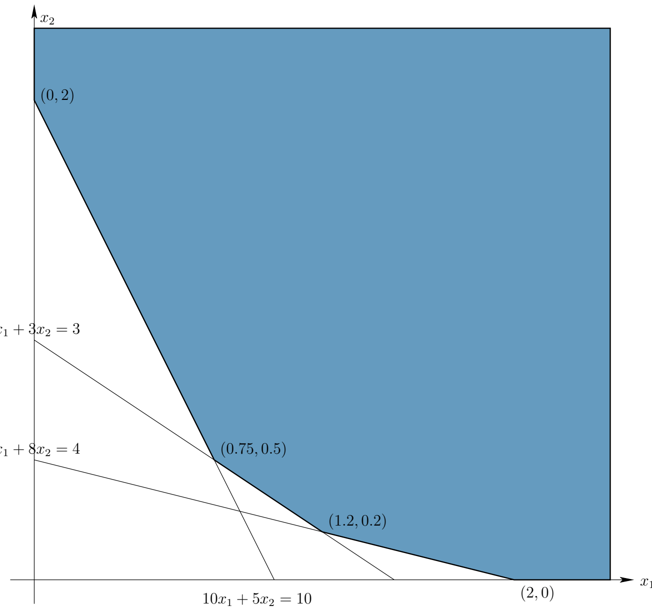

Figure 1: The region, shaded, for which all the feasible solutions to Question 4 must lie. The region has four extreme points: $(0, 2)$, $(0.75, 0.5)$, $(1.2, 0.2)$ and $(2, 0)$.

3. Find (graphically, or otherwise) the set of extreme points for the following linear programming problem:

$$\begin{aligned} \text{maximise} \quad & z = 2x_1 + 5x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \geq 2 \\ & x_1 + x_2 \leq 8 \\ & x_1 + x_2 \geq 3 \\ & 2x_1 + x_2 \leq 12 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

   Compute the value of the objective function at each extreme point. What is the optimal solution to the problem?

4. Recall the cost minimisation problem from Lecture 2, which we solved as question 1 on Question Sheet Zero.

$$\begin{aligned} \text{minimise} \quad & z = 40x_1 + 80x_2 \\ \text{subject to} \quad & 10x_1 + 5x_2 \geq 10 \\ & 2x_1 + 3x_2 \geq 3 \\ & 2x_1 + 8x_2 \geq 4 \\ & x_1, x_2 \geq 0. \end{aligned}$$

   The feasible region is given in Figure 1.

   (a) Write the problem in canonical form.

   (b) Determine all of the basic solutions of the problem in canonical form and interpret them in terms of the constraints of the problem in standard form.