MA30087/50087 - Question Sheet Zero

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2015/16 Semester I

Set: Problems Class, Thursday 1st October 2015.
Task: These are preliminary questions that we will aim to discuss in the problems class. There is no formal work set this week. Full solutions to this sheet will be available from Thursday 8th October 2015.

1. Recall the cost minimisation problem from Lecture 2. Our objective is to:

$$\begin{array}{ll} \text{minimise} & z = 40x_1 + 80x_2 \\ \text{subject to} & 10x_1 + 5x_2 \geq 10 \\ & 2x_1 + 3x_2 \geq 3 \\ & 2x_1 + 8x_2 \geq 4 \\ & x_1, x_2 \geq 0. \end{array}$$

   (a) Solve this linear programming problem using a graphical method.

   (b) Write down the vectors **b** and **c** and the matrix **A** when the problem is expressed as a standard maximisation problem.

2. Consider the linear programming problem:

$$\begin{array}{ll} \text{maximise} & z = 18x_1 + 6x_2 \\ \text{subject to} & 3x_1 + x_2 \leq 120 \\ & x_1 + 2x_2 \leq 160 \\ & x_1 \leq 35 \\ & x_1, x_2 \geq 0. \end{array}$$

   Solve the problem using a graphical method and comment upon your answer.

3. Consider the linear programming problem:

$$\begin{array}{ll} \text{maximise} & z = x_1 + 2x_2 + \frac{1}{2}x_3 \\ \text{subject to} & x_1 + x_2 \leq 4 \\ & x_1 + x_3 \geq 7 \\ & x_3 - x_2 \leq 1 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

   Show that there does not exist a feasible solution to the problem.

4. Consider the linear programming problem:

$$\begin{aligned} \text{maximise} \quad & z = 3x_1 + 4x_2 + x_3 \\ \text{subject to} \quad & 4x_1 + 2x_2 - x_3 = 5 \\ & x_1 + 3x_2 \geq 2 \\ & x_2 - x_3 \leq 3 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(a) Write down the vectors **b** and **c** and the matrix **A** when the problem is expressed as a standard maximisation problem.

(b) Suppose that the problem is now altered so that $x_1 \geq 1$. Write down an equivalent problem in standard maximisation form.

(c) Suppose that the constraints on the variables are that $x_1, x_2 \geq 0$ but $x_3 \in \mathbb{R}$ is no longer required to be non-negative. By considering $x_3 = w_3 - y_3$, where $w_3 \geq 0$ and $y_3 \geq 0$, write down an equivalent problem in standard maximisation form.