7/12/2015. PROBLEMS CLASS: CB1.11. Ⓐ

5. $\underline{s} = \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$ $\underline{d} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$ $\underline{c} = \begin{pmatrix} 50 & 60 & 30 \\ 60 & 40 & 20 \\ 40 & 70 & 30 \end{pmatrix}$

$\sum_{i=1}^{3} s_i = 16, \quad \sum_{j=1}^{3} d_j = 12.$

The problem is unbalanced. Add a "dump" to absorb the excess supply.

$\underline{s} = \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$ $\underline{d}' = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 4 \end{pmatrix}$ $\underline{c}' = \begin{pmatrix} 50 & 60 & 30 & 0 \\ 60 & 40 & 20 & 0 \\ 40 & 70 & 30 & 0 \end{pmatrix}$

Initial bfs using the matrix method:

$$
\begin{array}{c|cccc|c}
T_1 & 5 & 4 & 3 & 4 & u_i \\
\hline
8 & 2 \;{\scriptstyle 50} & 2-\eta \;{\scriptstyle 60} & \eta \;{\scriptstyle 30} & 4 \;{\scriptstyle 0} & 0 \\
5 & {\scriptstyle 60} & 2+\eta \;{\scriptstyle 40} & 3-\eta \;{\scriptstyle 20} & {\scriptstyle 0} & -20 \\
3 & 3 \;{\scriptstyle 40} & {\scriptstyle 70} & {\scriptstyle 30} & {\scriptstyle 0} & -10 \\
\hline
v_j & 50 & 60 & 40 & 0 &
\end{array}
$$

Change to objective:
-60η
$+30\eta$
-20η
$+40\eta$
$\overline{-20\eta}$

$\sum_{j=1}^{3} x_{1j} < s_1$

$\sum_{j=1}^{3} x_{2j} = s_2, \quad \sum_{j=1}^{3} x_{3j} = s_3$

Total cost: $\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} = (50\times 2) + (60\times 2) + (40\times 2) + (20\times 3) + (40\times 3) + (4\times 0)$
$= 480$

$\sum_{i=1}^{3} s_i u_i + \sum_{j=1}^{4} d_j v_j = (0\times 8) + (-20\times 5) + (-10\times 3)$
$+ (50\times 5) + (60\times 4) + (40\times 3) + (0\times 4) = 480.$

$[t_{ij}] = [c_{ij} - u_i - v_j] = \begin{bmatrix} 0 & 0 & -10 & 0 \\ 30 & 0 & 0 & 20 \\ 0 & 20 & \boxed{0} & 10 \end{bmatrix}$ Reduced cost of zero for a non-basic variable.

As $t_{13} < 0$, add x_{13} to the basis. Take $\eta = 2$ and push x_{12} out of the basis.

B

$\eta = 2,\ t_{13} = -10 \Rightarrow z = 480 - 2(10) = 460.$

T_2	5	4	3	4	u_i
8	2 50	60	2 30	4 0	0
5	60	4 40	1 20	0	−10
3	3 40	70	30	0	−10
v_j	50	50	30	0	460

$$[t_{ij}] = \begin{bmatrix} 0 & 10 & 0 & 0 \\ 20 & 0 & 0 & 10 \\ 0 & 30 & 10 & 10 \end{bmatrix}$$

As $[t_{ij}] \geq 0$ the dual solution is feasible and so the primal is optimal.

Initial BFS using the N-W corner method:

T_1	5	4	3	4	u_i
8	5 50	3 60	30	0	0
5	60	1 40	3 20	1 0	−20
3	40	70	30	3 0	−20
v_j	50	60	40	20	530

$z = (5\times50) + (3\times60) + (1\times40) + (3\times20) + (1\times0) + (3\times0) = 530$

$(0\times8) + (-20\times5) + (-20\times3) + (50\times5) + (60\times4) + (40\times3) + (20\times4) = 530.$

C.

4. Balanced transportation problem in matrix form has:

$$\underline{A} = \begin{array}{cc} & \begin{array}{ccc} \quad n \quad & \quad n \quad & \quad n \quad \end{array} \\ \begin{array}{c} m \\ \\ \\ n \end{array} & \begin{bmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 1 & \cdots & 1 \\ 1 & \cdots & 0 & 1 & \cdots & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 1 & \cdots & 0 & \cdots & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \leftarrow \underline{r}_1^T \\ \leftarrow \underline{r}_2^T \\ \quad \vdots \\ \leftarrow \underline{r}_m^T \\ \leftarrow \underline{r}_{m+1}^T \\ \quad \vdots \\ \leftarrow \underline{r}_{m+n}^T \end{array}$$

only one non-zero value.

Note: $\sum_{i=1}^{m} r_i = \sum_{j=1}^{n} r_j = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \Rightarrow \text{Rank}(\underline{A}) < m+n.$

Drop the first row and consider $\sum_{i=1}^{m+n-1} \lambda_i \underline{r}_i = \underline{0}_{mn}$

We'll show $\underline{r}_i$ s are independent i.e. $\lambda_1 = \cdots = \lambda_{m+n-1} = 0$.

nth entry: $\left(\sum_{i=1}^{m+n-1} \lambda_i r_i\right)_n = \lambda_1 = 0$ nth column

$\left(\sum_{i=1}^{m+n-1} \lambda_i r_i\right)_{2n} = \lambda_2 = 0$ 2nth col.

$\vdots$

$\left(\sum_{i=1}^{m+n-1} \lambda_i r_i\right)_{mn} = \lambda_m = 0$ mnth column.

But, $\left(\sum_{i=1}^{m+n-1} \lambda_i r_i\right)_1 = \lambda_1 + \lambda_{m+1} = 0 \Rightarrow \lambda_{m+1} = 0$ 1st column.

$\vdots$

$\left(\sum_{i=1}^{m+n-1} \lambda_i r_i\right)_{n-1} = \lambda_1 + \lambda_{m+n-1} = 0 \Rightarrow \lambda_{m+n-1} = 0.$ nth column.

$\Rightarrow \text{Rank}(\underline{A}) = m+n-1.$