5/11/2015. PROBLEMS CLASS: 3WN2·1. Ⓐ.

QUESTION SHEET FOUR.

5.

T_1	z	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	3	1 [4]	−4	1	0	0	4
s_2	0	1	−1	−1	0	1	$\emptyset$	10
s_3	0	1	−2	6 [9/6]	0	0	$\emptyset$	9
	1	1	−2	−1	0	0	0	0

INTRODUCE EITHER x_2 OR x_3 INTO THE BASIS.
ADDING x_2 WILL PUSH OUT s_1 AND ADD $\underset{r_j}{2}(\underset{a_j}{4}) = 8$ TO z

ADDING x_3 WILL PUSH OUT s_3 AND ADD $1(9/6) = 9/6$ TO z.
WE'LL ADD x_2 TO THE BASIS.

T_2	z	x_1	x_2	x_3	s_1	s_2	s_3	
x_2	0	3	1	−4	1	0	0	4
s_2	0	4	0	−5	1	1	0	14
s_3	0	7	0	−2	2	0	1	17
	1	7	0	−9	2	0	0	8

} REPRESENTS SOLUTIONS TO $\underline{A}\,\underline{x} = \underline{b}$

$z = 8 - 7x_1 + 9x_3 - 2s_1$. CONSIDER ADDING x_3 TO THE BASIS, KEEPING x_1 AND s_1 NON-BASIC. LET $x_3 = M > 0$, $x_1 = s_1 = 0$ SOLUTIONS TO $\underline{A}\,\underline{x} = \underline{b}$ WITH $x_1 = s_1 = 0$ SATISFY:

$$x_2 - 4x_3 = 4$$
$$s_2 - 5x_3 = 14$$
$$s_3 - 2x_3 = 17$$

IF $x_3 = M > 0$ THEN $x_2 = 4 + 4M$, $s_2 = 14 + 5M$, $s_3 = 17 + 2M$.
THUS, $(0, 4+4M, M, 0, 14+5M, 17+2M)$ FEASIBLE $\forall\ M \geq 0$.

THE CORRESPONDING VALUE OF $z = 8 + 9M$.
AS $M \to \infty$ $z \to \infty$.
IF WE WANT A SOLUTION WITH $z > 1000$ THEN WE REQUIRE
$8 + 9M > 1000$ i.e. $M > \frac{1000-8}{9} = 110.22$.

TAKING $M = 111$ GIVES $(0, 448, 111, 0, 569, 239)$, $z = 1007$.

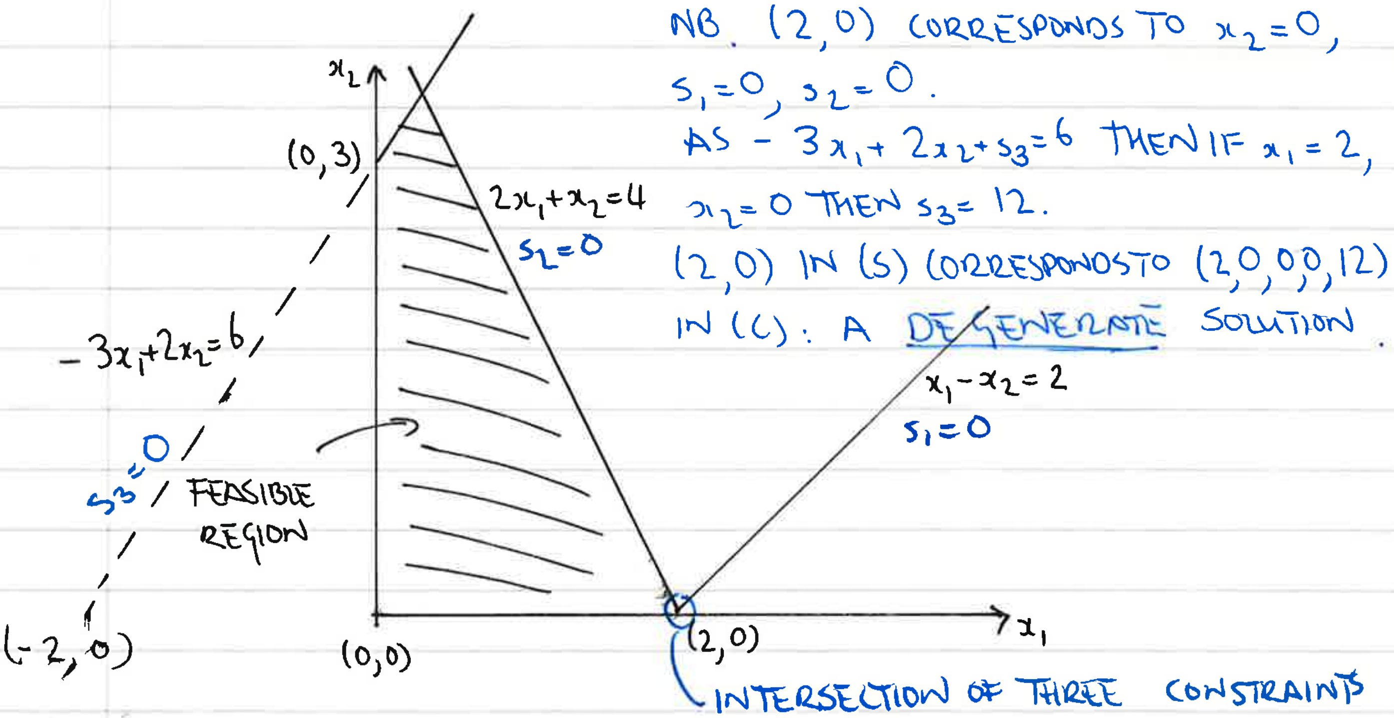
QUESTION SHEET FIVE.

3. max $z = 3x_1 + x_2$
subject to
(S)
$$x_1 - x_2 \le 2$$
$$2x_1 + x_2 \le 4$$
$$-3x_1 + 2x_2 \le 6$$
$$x_1, x_2 \ge 0$$

$\rightsquigarrow$

maximise $z = 3x_1 + x_2$
(C)
$$x_1 - x_2 + s_1 = 2$$
$$2x_1 + x_2 + s_2 = 4$$
$$-3x_1 + 2x_2 + s_3 = 6$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0$$

NB. $(2,0)$ CORRESPONDS TO $x_2 = 0$, $s_1 = 0$, $s_2 = 0$.
AS $-3x_1 + 2x_2 + s_3 = 6$ THEN IF $x_1 = 2$, $x_2 = 0$ THEN $s_3 = 12$.
$(2,0)$ IN (S) CORRESPONDS TO $(2,0,0,0,12)$ IN (C): A DEGENERATE SOLUTION.



MAXIMUM IS $z = 6$ AT $(2,0)$.

FEASIBLE REGION BOUNDED SO FINITE SOLUTION EXISTS WHICH CAN BE FOUND AT AN EXTREME POINT.

$(-2,0)$.

$(0,0)$	$z = 0$	$(2,0)$	$z = 6$
$(0,3)$	$z = 3$	$(2/7, 24/7)$	$z = 4\,2/7$

(WE'LL COVER THE SIMPLEX PART NEXT WEEK!)

T_1	z	x_1	x_2	s_1	s_2	s_3	
s_1	0	1 [2]	-1	1	0	0	2
s_2	0	2 [2]	1 [4]	0	1	0	4
s_3	0	-3	2 [3]	0	0	1	6
	1	-3	-1	0	0	0	0

$z = 0 + 3x_1 + x_2$. CONSIDER ADDING EITHER x_1 OR x_2 TO THE BASIS. TAKING $x_1 = 2$ WILL ADD 6 TO z
TAKING $x_2 = 3$ WILL ADD 3 TO z.

NOTE IF $x_1 = 2$ THEN BOTH s_1 AND s_2 GO TO ZERO. WE'LL GET A DEGENERATE SOLUTION.

WANT TO MAINTAIN A BASIS OF SIZE $m=3$ SO HAVE TO DECIDE WHICH OF s_1, s_2 TO PUSH OUT. LET'S PUSH OUT s_1.

T_2	z	x_1	x_2	s_1	s_2	s_3	
x_1	0	1	-1	1	0	0	2
s_2	0	0	3 [0]	-2	1	0	0
s_3	0	0	-1	3	0	1	12
	1	0	-4	3	0	0	6

BASIC.

THIS IS THE POINT $(2, 0, 0, \boxed{0}, 12)$ WITH $z = 6$.
WE KNOW THIS IS OPTIMAL. HOWEVER UNDER THIS BASIS
$z = 6 + 4x_2 - 3s_1$ SO WE CAN'T CONFIRM IT.

LET'S ADD x_2 TO THE BASIS, PUSHING OUT s_2.

D

T_3	z	x_1	x_2	s_1	s_2	s_3	
x_1	0	1	0	$1/3$	$1/3$	0	2
x_2	0	0	1	$-2/3$	$1/3$	0	0
s_3	0	0	0	$7/3$	$1/3$	1	12
	1	0	0	$1/3$	$4/3$	0	6

BFS IS $(2, \underbrace{\boxed{0}}_{\text{BASIC}}, 0, 0, 12)$ WITH $z = 6 - \frac{1}{3}s_1 - \frac{4}{3}s_2 \leq 6$

I CAN NOW IDENTIFY THIS AS THE OPTIMAL SOLUTION.

DEGENERATE SOLUTIONS CAN POSE PROBLEMS FOR THE SIMPLEX ALGORITHM AND IF YOU'RE NOT CAREFUL YOU CAN CYCLE (i.e. RETURN TO SAME SOLUTION)

THERE'S AN EXAMPLE OF THIS ON QUESTION 4.