29/10/2015. PROBLEMS CLASS: 3W2.1. (A)

4. IN CANONICAL FORM WE HAVE:

$$\begin{array}{ll} \text{maximum} & z = 2x_1 - 3x_2 + x_3 \\ \text{subject to} & 3x_1 + 6x_2 + x_3 + s_1 = 6 \\ & 4x_1 + 2x_2 + x_3 + s_2 = 4 \\ & x_1 - x_2 + x_3 + s_3 = 3 \\ & x_1, x_2, x_3, s_1, s_2, s_3 \geq 0 \end{array}$$

NOTE $\underline{b} = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix} \geq \underline{0}$ SO THAT BASIS OF $\{s_1, s_2, s_3\}$ (SLACK VARIABLES) WILL BE A BASIC FEASIBLE SOLUTION.

T_1	z	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	3	6	1	1	0	0	6
s_2	0	4	2	1	0	1	0	4
s_3	0	1	-1	1	0	0	1	3
	1	-2	3	-1	0	0	0	0

BASIS

$\underline{x}_B + \underline{B}^{-1}\underline{N}\,\underline{x}_N = \underline{B}^{-1}\underline{b}$ HERE $\underline{B} = \underline{I}_3$.

REPRESENTATION OF $z - (\underline{c}_N^T - \underline{c}_B^T \underline{B}^{-1}\underline{N})\underline{x}_N = \underline{c}_B^T \underline{B}^{-1}\underline{b}$

$$z = \underline{c}_B^T \underline{B}^{-1}\underline{b} + \overbrace{(\underline{c}_N^T - \underline{c}_B^T \underline{B}^{-1}\underline{N})}^{\text{REDUCED COSTS}}\underline{x}_N$$

VALUE OF THE OBJECTIVE FUNCTION AT BFS ASSOCIATED TO $\underline{B}$.

INITIAL BFS $\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \\ (0, & 0, & 0, & 6, & 4, & 3)^T \end{array}$ WITH

$$z = 0 + 2x_1 - 3x_2 + x_3$$

TAKING EITHER $x_1 > 0$ OR $x_3 > 0$ WILL INCREASE z. WE WANT TO CHANGE THE BASIS TO INCLUDE EITHER x_1 OR x_3 WHILST MAINTAINING FEASIBILITY.

WHAT HAPPENS IF WE ADD x_1 TO THE BASIS?

x_2 AND x_3 REMAIN NON-BASIC SO EQUAL TO ZERO.

$$\begin{aligned} 3x_1 + s_1 &= 6 \\ 4x_1 + s_2 &= 4 \\ x_1 + s_3 &= 3 \end{aligned} \quad \text{equivalently:} \quad \begin{aligned} x_1 + \tfrac{1}{3}s_1 &= 2 \\ x_1 + \tfrac{1}{4}s_2 &= 1 \\ x_1 + s_3 &= 3 \end{aligned}$$

TAKE $x_1 = \min\{2, 1, 3\} = 1$. THIS REQUIRES $s_2 = 0$. IF $x_1 > 1$ THEN $s_2 < 0$ SO THE PROBLEM IS NOT FEASIBLE.

REDUCED COST FOR x_1

TAKING $x_1 = 1$ WILL ADD $2(1)$ TO z.

WHAT HAPPENS IF WE ADD x_3 TO THE BASIS?

x_1 AND x_2 REMAIN NON-BASIC SO EQUAL TO ZERO.

$$\begin{aligned} x_3 + s_1 &= 6 \\ x_3 + s_2 &= 4 \\ x_3 + s_3 &= 3 \end{aligned}$$

TAKE $x_3 = \min\{6, 4, 3\} = 3$. THIS REQUIRES $s_3 = 0$. IF $x_3 > 3$ THEN $s_3 < 0$ SO NOT FEASIBLE.

REDUCED COST FOR x_3

TAKING $x_3 = 3$ WILL ADD $1(3) = 3$ TO z

⊕ ADD x_3 TO THE BASIS, REMOVING s_3 ⊕

(c).

PERFORM ROW OPERATIONS ON T_1 TO BRING x_3 INTO THE BASIS.

- 3^{rd} EQUATION MULTIPLY BY 1
- 1^{st} EQUATION: SUBTRACT 3^{rd} EQUATION.
- 2^{nd} EQUATION: SUBTRACT 3^{rd} EQUATION
- 4^{th} EQUATION: ADD 3^{rd} EQUATION.

T_2	z	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	2 [3/2]	7	0	1	0	-1	3
s_2	0	3 [1/3]	3	0	0	1	-1	1
x_3	0	1 [3/1]	-1	1	0	0	1	3
	1	-1	2	0	0	0	1	3

NEW BFS IS $(0, 0, 3, 3, 1, 0)^T$

$$z = 3 + x_1 - 2x_2 - s_3$$

SOLUTION IS NOT OPTIMAL: ANY SOLUTION WITH $x_1 > 0$ WILL INCREASE z. ADD x_1 TO THE BASIS.

x_2, s_3 REMAIN NON-BASIC SO EQUAL TO ZERO.

$$\begin{aligned} 2x_1 + s_1 &= 3 \\ 3x_1 + s_2 &= 1 \\ x_1 + x_3 &= 3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_1 + \tfrac{1}{2} s_1 &= 3/2 \\ x_1 + \tfrac{1}{3} s_2 &= 1/3 \\ x_1 + x_3 &= 3 \end{aligned}$$

TAKE $x_1 = \min\{3/2, 1/3, 3\}$ WHICH REMOVES s_2. IT WILL ADD $1(1/3)$ TO z.

(D)

BASIS SWITCH

T_3	z	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	0	5	0	1	$-2/3$	$-1/3$	$7/3$
x_1	0	1	1	0	0	$1/3$	$-1/3$	$1/3$
x_3	0	0	-2	1	0	$-1/3$	$4/3$	$8/3$
	1	0	3	0	0	$1/3$	$2/3$	$10/3$

COLUMN SUMMARISES ROW OPERATIONS
e.g. ADD $-2/3$ OF 2^{nd} TO 1^{st}.

THE BFS $\underline{x}^* = (1/3, 0, 8/3, 7/3, 0, 0)^T$ IS OPTIMAL AS

$$z = \frac{10}{3} - 3x_2 - \frac{1}{3}s_2 - \frac{2}{3}s_3 \leq \frac{10}{3} = z(\underline{x}^*).$$

AS $x_2, s_2, s_3 \geq 0$.