3/12/2015. LECTURE 21: 3WN2.1.

For the transportation algorithm the CS conditions are equivalent to

$$x_{ij} > 0 \Rightarrow u_i + v_j = c_{ij} \quad \text{or} \quad u_i + v_j < c_{ij} \Rightarrow x_{ij} = 0$$

STRATEGY.

① Find a bfs with at most $m+n-1$ non-zero values of x_{ij}

[PRIMAL FEASIBILITY]

② Use asymmetric CS conditions, write down $m+n-1$ equations

$$u_i + v_j = c_{ij}$$

$m+n$ unknowns. Set (for example) $u_1 = 0$ and solve.

[SATISFYING CS CONDITIONS]

③. If u_i, v_j satisfy $u_i + v_j \leq c_{ij}$ $\forall i, j$ optimal solution found

[DUAL FEASIBILITY]

If not, move to another bfs of btp and repeat.

6.3.1 FINDING AN INITIAL BFS: THE NORTH-WEST CORNER METHOD.

Represent shipments from S_i to D_j as an $m \times n$ matrix

	d_1	$\cdots$	d_n	
s_1	x_{11}	$\cdots$	x_{1n}	$\sum_{j=1}^{n} x_{1j} = s_1$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
s_m	x_{m1}	$\cdots$	x_{mn}	$\sum_{j=1}^{n} x_{mj} = s_m$
	$\sum_{i=1}^{m} x_{i1} = d_1$	$\cdots$	$\sum_{i=1}^{m} x_{in} = d_n$	

ALLOCATE: at most $m+n-1$ non-zero values to x_{ij} such that:

- Each row sums to required supply
- Each column sums to desired demand

NORTH-WEST CORNER METHOD.

1. START WITH N-W CORNER (i.e. x_{11})
2. ALLOCATE MAXIMUM FEASIBLE AMOUNT CONSISTENT WITH ROW AND COLUMN SUM REQUIREMENT.
   [e.g. $x_{11} = \min\{s_1, d_1\}$]
3. MOVE ONE CELL TO RIGHT IF ANY ROW REQUIREMENT LEFT, OTHERWISE MOVE ONE CELL DOWN. IF ALL REQUIREMENTS MET, STOP. OTHERWISE, GO TO STEP (2).

6.3.2 EXAMPLE OF THE ALGORITHM IN ACTION.

CONSIDER THE PROBLEM WITH COSTS $\underline{c}$, SUPPLY $\underline{s}$ AND DEMAND $\underline{d}$,

$\sum_{i=1}^{3} s_i = \sum_{j=1}^{4} d_j = 200$: the problem is balanced.

$$\underline{c} = \begin{pmatrix} 13 & 11 & 18 & 17 \\ 2 & 14 & 10 & 1 \\ 5 & 8 & 18 & 11 \end{pmatrix} \quad \underline{s} = \begin{pmatrix} 80 \\ 100 \\ 20 \end{pmatrix} \quad \underline{d} = \begin{pmatrix} 55 \\ 70 \\ 35 \\ 40 \end{pmatrix}$$

demands

T_1	55	70	35	40
80	55	25		
100		45	35	20
20				20

supplies

x_{11} + $(n-1)$ steps across
$(m-1)$ steps down
$1 + (n-1) + (m-1) = m+n-1$

Checking if bfs is optimal: solve the asymmetric CS conditions to find a proposed dual solution. If this solution is feasible, we have reached optimality.

(c).

T_1	55		70		35		40		u_i	
80	55	[13]	25	[11]		[18]		[17]	0	
100		[2]	45	[14]	35	[10]	20	[1]	3	$u_2+v_2=14$
20		[5]		[8]		[18]	20	[11]	13	$u_3+v_4=13$
v_j	13		11		7		-2		2210	
	$u_1+v_1=13$		$u_1+v_2=11$		$u_2+v_3=10$		$u_2+v_4=1$			

$z = (55\times13)+(25\times11)+(45\times14)+(35\times10)+(20\times1)+(20\times11)$

$(= (0\times80)+(3\times100)+(13\times20)+(13\times55)+(11\times70)+(7\times35)+(-2\times40))$

Solve $c_{ij} = u_i + v_j$ for $m+n-1$ basic x_{ij} (here $x_{ij} \neq 0$). Compute $t_{ij} = c_{ij} - u_i - v_j$. If $t_{ij} \geq 0 \ \forall i, j$ solution is optimal (as dual feasible)

$18-0-7$ $\quad$ $17-0-(-2)$

$$[t_{ij}] = \begin{bmatrix} 0 & 0 & 11 & 19 \\ -14 & 0 & 0 & 0 \\ -21 & -16 & -2 & 0 \end{bmatrix}$$

↑ WORST OFFENDER.

= HERE.

Solution is not optimal. The $[t_{ij}]$ represent the reduced costs

$[\text{"} \; \underline{c}_N^T - \underline{c}_B^T B^{-1} \underline{N} = \underline{c}_N^T - \underline{y}^T \underline{N} \;]$ $\quad$ $\underline{y} = \underline{c}_B^T \underline{B}^{-1}$ dual solution.

Minimisation problem so want reduced costs positive for optimal.

Moving to a new bfs.

x_{31} is the "worst offender" in sense of smallest value of t_{ij}. Set $x_{31} = \eta > 0$ and perform a PERTURBATION LOOP

T_1	55	70	35	40
80	$55-\eta$	$25+\eta$		
100		$45-\eta$	35	$20+\eta$
20	η			$20-\eta$

LOOP KEEPS EACH ROW SUM AND EACH COLUMN SUM UNCHANGED.