Example of the two-phase artificial variables method; question 4 of Question Sheet Six

We begin by adding in relevant slack variables:

$$\begin{array}{lr} \text{maximise} & 2x_1 + x_2 + 3x_3 \\ \text{subject to} & x_1 - 3x_2 + \frac{1}{2}x_3 = 6 \\ & -x_1 + 8x_2 + \frac{1}{2}x_3 + s_1 = 4 \\ & -3x_1 + 4x_2 + \frac{1}{2}x_3 - s_2 = 1 \\ & x_1, x_2, x_3, s_1, s_2 \geq 0. \end{array}$$

Then the first and third equations also require an artificial variable, and our first phase is to maximise $z' = -u_1 - u_2$.

T_1	z'	z	x_1	x_2	x_3	s_1	s_2	u_1	u_2	
u_1	0	0	1	-3	$\frac{1}{2}_{[12]}$	0	0	1	0	6
s_1	0	0	-1	$8_{[\frac{1}{2}]}$	$\frac{1}{2}_{[8]}$	1	0	0	0	4
u_2	0	0	-3	$4_{[\frac{1}{4}]}$	$\frac{1}{2}_{[2]}$	0	-1	0	1	1
II	0	1	-2	-1	-3	0	0	0	0	0
I	1	0	2	-1	-1	0	1	0	0	-7

We should introduce x_3 for u_2. The new tableau is:

T_2	z'	z	x_1	x_2	x_3	s_1	s_2	u_1	u_2	
u_1	0	0	$4_{[\frac{5}{4}]}$	-7	0	0	$1_{[5]}$	1	$-$	5
s_1	0	0	$2_{[\frac{3}{2}]}$	4	0	1	$1_{[3]}$	0	$-$	3
x_3	0	0	-6	8	1	0	-2	0	$-$	2
II	0	1	-20	23	0	0	-6	0	$-$	6
I	1	0	-4	7	0	0	-1	0	$-$	-5

The next step is to introduce x_1 for u_1. This gives tableau T_3.

T_3	z'	z	x_1	x_2	x_3	s_1	s_2	u_1	u_2	
x_1	0	0	1	$-\frac{7}{4}$	0	0	$\frac{1}{4}_{[5]}$	$-$	$-$	$\frac{5}{4}$
s_1	0	0	0	$\frac{15}{2}_{[\frac{1}{15}]}$	0	1	$\frac{1}{2}_{[1]}$	$-$	$-$	$\frac{1}{2}$
x_3	0	0	0	$-\frac{5}{2}$	1	0	$-\frac{1}{2}$	$-$	$-$	$\frac{19}{2}$
II	0	1	0	-12	0	0	-1	$-$	$-$	31
I	1	0	0	0	0	0	0	$-$	$-$	0

The first phase is complete, we have a basic feasible solution to the new problem, and so we enter phase II and drop the artificial variables. The next step is to introduce s_2 for s_1.

T_4	z	x_1	x_2	x_3	s_1	s_2	
x_1	0	1	$-\frac{11}{2}$	0	$-\frac{1}{2}$	0	1
s_2	0	0	15	0	2	1	1
x_3	0	0	5	1	1	0	10
	1	0	3	0	2	0	32

Every entry in the final row is positive, so this is an optimal solution, and corresponds to the solution $x_1 = 1, x_2 = 0, x_3 = 10$ (and $s_1 = 0, s_2 = 1$), and $z = 32$.