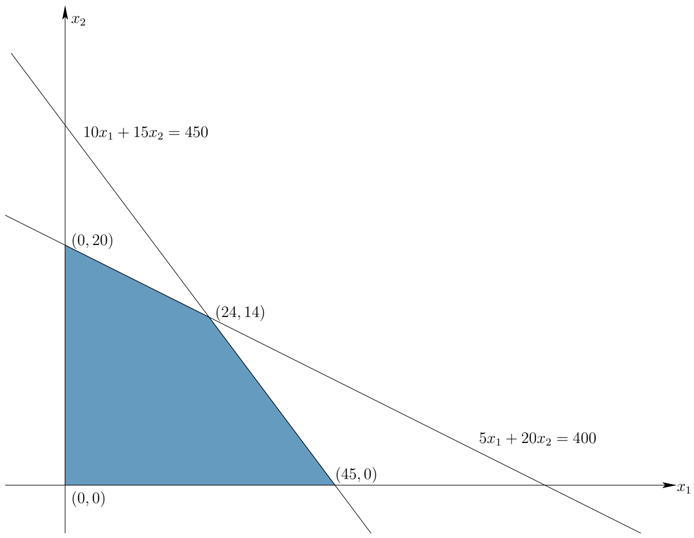

Figure 1: The region for which all the feasible solutions to Example 1 must lie. The region has four extreme points: $(0, 0)$, $(0, 20)$, $(24, 14)$ and $(45, 0)$.

Example 1 *A furniture company plans to make two products, oak chairs and oak tables, from its available resources. It has 400 board feet of oak available and 450 hours of labouring time. A chair takes 10 hours to produce, utilising 5 board feet whilst each table takes 15 hours to produce requiring 20 board feet. Both items are sold at a profit, tables earning twice the profit of chairs. Let* x_1 *denote the number of chairs produced and* x_2 *the number of tables. Our aim is to*

$$\begin{array}{lrcl} \textit{maximise} & z & = & x_1 + 2x_2 \\ \textit{subject to} & 5x_1 + 20x_2 & \leq & 400 \\ & 10x_1 + 15x_2 & \leq & 450 \\ & x_1, x_2 & \geq & 0 \end{array}$$

This is a linear programming problem with two variables and two constraints.[1] *As this is a problem in* $\mathbb{R}^2$ *we can solve the problem graphically.*

In Figure 1 we illustrate the ***feasible region****, that is the set of points with coordinates* (x_1, x_2) *that satisfy all of the constraints.*

[1] Strictly there are four constraints but, as we shall shortly see, positivity of variables is a usual requirement.

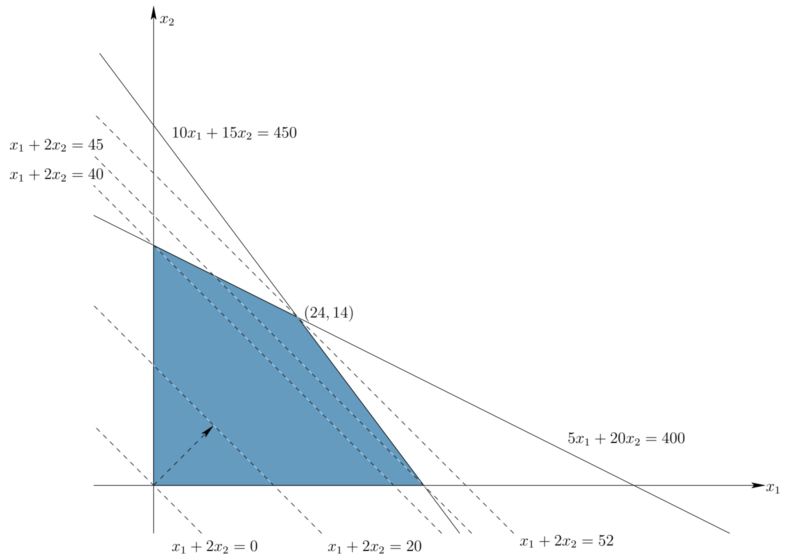

Figure 2: Translating the objective function of Example 1 through the feasible region. The maximum of 52 occurs at the extreme point $(24, 14)$.

The objective is to maximise the linear function $z = x_1 + 2x_2$*. Figure 2 illustrates how we do this. If we fix* $z = 0$ *we can see that the objective function passes through the origin. Increasing the value of* z *corresponds to translating the objective function through the feasible region (in a northeast direction). For example, Figure 2 shows the objective function for increasing values of* z*. Note that when* $z = 40$ *we obtain the maximum feasible value of* x_2 *and when* $z = 45$ *we obtain the maximum feasible value of* x_1*. Continuing to increase* z *we see that* ***the maximum is achieved at the extreme point*** $(24, 14)$ *where* $z = 52$*.*