MA20226 - Question Sheet Six

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2011/12 Semester I

Set: Tutorials of Week 10, commencing Monday 5th December 2011.
Due in: Tutorials of Week 11, commencing Monday 12th December 2011. You should write both your name and tutorial group on your work. If you are unable to make your practical then you should ensure that you hand the work to me personally *before* the practical is held; my office is 4W4.10.
Task: Attempt questions 1-3; questions 4-6 are additional questions and may be used for tutorial discussion.

1. Suppose that we are going to gather a sample of twenty independent observations from a $N(\mu, \sigma^2 = 1.0)$ distribution in order to make inferences about μ. Suppose we are interested in the null hypothesis $H_0 : \mu = 10$.

   (a) State the two-tailed alternative hypothesis and find the appropriate critical values for a test of significance 0.05.

      i. What is the p-value of this test when we observe a sample mean $\bar{x} = 10.41$?
      ii. Evaluate the power at $\mu = 10.5$.

   (b) State the upper one-tailed alternative hypothesis and find the appropriate critical values for a test of significance 0.05.

      i. What is the p-value of this test when we observe a sample mean $\bar{x} = 10.41$?
      ii. Evaluate the power at $\mu = 10.5$.

2. The Food and Nutrition Board of the National Academy of Sciences (USA) suggests that the RDA of iron for adult females under the age of 51 is 18mg. The following iron intakes, in mg, during a 24-hour period were observed for 45 randomly selected adult females in this age group:

15.0	18.1	14.4	14.6	10.9	18.1	18.2	18.3	15.0
16.0	12.6	16.6	20.7	19.8	11.6	12.8	15.6	11.0
15.3	9.4	19.5	18.3	14.5	16.6	11.5	16.4	12.5
14.6	11.9	12.5	18.6	13.1	12.1	10.7	17.3	12.4
17.0	6.3	16.8	12.5	16.3	14.7	12.7	16.3	11.5

   You may assume that each observation represents the realisation of a $N(\mu, \sigma^2)$ random quantity and use the facts that $\sum_{i=1}^{45} x_i = 660.6$, $\sum_{i=1}^{45} x_i^2 = 10115.88$, $\chi^2_{44,0.975} = 27.574$ and $\chi^2_{44,0.025} = 64.201$.

   (a) Evaluate a 95% confidence interval for σ^2.

(b) Test the hypotheses $H_0 : \sigma^2 = 10$ versus $H_1 : \sigma^2 \neq 10$ using a 5% significance level. Express the p-value of this test as a probability statement involving the test statistic (the tables which you have will not give the necessary quantiles to evaluate this p-value, but you could investigate using R to find it).

3. To assess the difficulty associated with the exams for two courses, Really Interesting Stats (A) and Yet More Fascinating Stats (B), ten students were given three questions for both of the courses and were given unlimited time to complete them. The following times in minutes were recorded

Student	1	2	3	4	5	6	7	8	9	10
Course A	124	98	126	127	130	116	108	118	106	122
Course B	107	96	115	120	102	118	102	117	111	113

Use a t-test to test the hypothesis that the two course exams have the same difficulty on average in terms of completion time. Comment on the assumptions you make and whether they seem appropriate.

4. Let $X_1, \ldots, X_n$ be independent and identically distributed as $N(\mu, \sigma^2)$ where σ^2 is known. Consider the hypothesis test

$$H_0\colon\ \mu = \mu_0 \quad \text{versus} \quad H_1\colon\ \mu \neq \mu_0$$

with critical region $C = \{(x_1, \ldots, x_n) : \Lambda(x_1, \ldots, x_n) \leq k\}$ where $\Lambda(x_1, \ldots, x_n) = L(\mu_0)/L(\overline{x})$, the ratio of the likelihood evaluated at μ_0 and $\overline{x}$, and k is a constant.

(a) Using the identity

$$\sum_{i=1}^{n}(x_i - \mu_0)^2 \quad = \quad \sum_{i=1}^{n}(x_i - \overline{x})^2 + n(\overline{x} - \mu_0)^2$$

show that

$$-2\log\Lambda(x_1, \ldots, x_n) \quad = \quad \frac{n(\overline{x} - \mu_0)^2}{\sigma^2}$$

and that the critical region may be expressed as

$$C \quad = \quad \{(x_1, \ldots, x_n) : -2\log\Lambda(x_1, \ldots, x_n) \geq k_1\}$$

for some constant k_1.

(b) Under H_0, what is the distribution of $-2\log\Lambda(X_1, \ldots, X_n)$? Hence, give the critical region for a test with significance level α.

(c) Explain why this critical region is equivalent to the test with critical region $\{(x_1, \ldots, x_n) : |\overline{x} - \mu_0| \geq \frac{\sigma}{\sqrt{n}} z_{(1-\frac{\alpha}{2})}\}$ where, for $Z \sim N(0, 1)$, $P(Z \leq z_{(1-\frac{\alpha}{2})}) = 1 - \frac{\alpha}{2}$.

5. In an experiment to compare two varieties of bean, six pairs of plots were sown with beans of the two types. Plots in the same pair were similar in soil, drainage etc but there were differences between pairs. The following yields (in pounds) were observed:

Plot	1	2	3	4	5	6
Variety A	52	72	71	86	48	81
Variety B	45	66	68	80	50	72

Working with the paired differences $\{D_i\}$, estimate μ_D and σ_D^2, and perform a paired t-test to test whether there is a difference between the varieties.

6. An experiment was carried out to test a new method for reducing faults on telephone lines. Fourteen matched pairs of areas were used, with one member of the pair receiving the new method, or test, and the other member acting as a control. The following table shows the fault rates for the test areas and for the control areas and the difference in their fault rates.

Control	Test	Difference
88	676	−588
570	206	364
605	230	375
617	256	361
653	280	373
2913	433	2480
924	337	587
286	466	−180
1098	497	601
982	512	470
2346	794	1552
321	428	−107
615	452	163
519	512	7

If d_i denotes the ith difference, then $\sum_{i=1}^{14} d_i = 6458$ and $\sum_{i=1}^{14} d_i^2 = 10444556$. Test, at the 5% level, the hypothesis that the mean fault rates are the same against the hypothesis that they differ. What assumptions have you made in performing the test, and to what extent do they seem justified here?