MA20226 - Feedback on Question Sheet Three

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Overall, this was the best sheet so far so well done to all of you who handed it in.

1. Ultimately this question was well done and the only point I really want to make is to be on the look out for savings in the algebra. For example, as we are dealing with a probability distribution, we know that

$$\int_0^\infty \frac{x^3}{6\theta^4}\exp\left(-\frac{x}{\theta}\right)dx \quad = \quad 1.$$

So, when calculating $E(X_i\,|\,\theta)$ you only need to use integration by parts once:

$$E(X_i\,|\,\theta) \quad = \quad \left[-\frac{x^4}{6\theta^3}\exp\left(-\frac{x}{\theta}\right)\right]_0^\infty + \int_0^\infty \frac{4x^3}{6\theta^3}\exp\left(-\frac{x}{\theta}\right)dx \;=\; 4\theta.$$

A similar trick applies when calculating $E(X_i^2\,|\,\theta)$. As you now also know that

$$\int_0^\infty \frac{x^4}{6\theta^4}\exp\left(-\frac{x}{\theta}\right)dx \quad = \quad 4\theta$$

you only need to use integration by parts once when finding $E(X_i^2\,|\,\theta)$:

$$E(X_i^2\,|\,\theta) \quad = \quad \left[-\frac{x^5}{6\theta^3}\exp\left(-\frac{x}{\theta}\right)\right]_0^\infty + \int_0^\infty \frac{5x^4}{6\theta^3}\exp\left(-\frac{x}{\theta}\right)dx \;=\; 5\theta\times 4\theta.$$

It will not surprise you that this generalises for higher moments: it is straightforward to show that for $r = 1, 2, 3, \ldots$

$$E(X_i^r\,|\,\theta) \quad = \quad (r+3)\theta E(X_i^{r-1}\,|\,\theta).$$

2. By and large, this question was also nicely done. As the solution sheet shows, the really nice answers involved looking at the intervals of possible values the mean, median and once-trimmed mean could take as x varied between 0 and 30. The interval was largest for the mean and smallest for the median illustrating the robustness. Additionally, the mean was affected by all values of x whilst the median and once-trimmed mean were unaffected once x was sufficiently extreme. That is, not a central value for the median and either the largest or smallest value for the once-trimmed mean.

3. Almost uniformly this was done correctly. You'll notice that you are extensively using the result on Question Sheet Four.