MA20226 - Feedback on Question Sheet One

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1. The notion of an intuitive estimator caused some difficulty. What I was looking for here was for you to match the interpretation of the parameter with a sample equivalent. For example, if $X \sim Bin(n, p)$ then p is the probability of a success and X the number of successes in n trials. Thus, X/n is the proportion of successes in n trials which mirrors p. Suppose that we take a sample of m (as we've used n as a known binomial parameter) iid observations $X_1, \ldots, X_m$ from $Bin(n, p)$ and form $\overline{X} = \frac{1}{m}\sum_{i=1}^{n} X_i$. Then a natural, or intuitive, estimator of p is $\overline{X}/n$, the average number of successes in n trials. Also, some of the terminology used was very loose, for example writing a sample space equal to $(0, \infty)$ when you meant $0, 1, 2, \ldots$.

2. The solution has two stages. Firstly, you need to use the properties of normal distributions (see the solution sheet) to deduce that the distribution of $\overline{X}$ is normal. Secondly, you need to find the parameters of the normal distribution associated to $\overline{X}$, that is $E(\overline{X} \mid \mu, \sigma^2)$ and $Var(\overline{X} \mid \mu, \sigma^2)$. Most people tended to do the second stage but a large number omitted stage one.

3. Overall, the question was well done.

   (a) Be careful, particularly in an exam, that you don't throw away marks by not being explicit in your answers. As a general comment, you should try to write a few more words to link steps in proofs and also when concluding calculations. In this question, the vast majority correctly calculated the likelihood function but then didn't explicitly explain why it was sufficient to know $\overline{x}$ and n to evaluate it. You need a concluding line something like "Hence, we only need $\overline{x}$ and n and not $x_1, \ldots, x_n$ to compute $L(\tau)$."

   (b) Generally, this was nicely done but be aware of making silly mistakes with the algebra. For example, a common error when constructing the likelihood was to take the $\frac{1}{\tau}$ outside of the product without raising it to the power of n.

   (c) In this case $\tilde{\tau} = \overline{X}$. To show it's unbiased you need to argue that, for each i, $E(X_i \mid \tau) = \tau$ so that $E(\overline{X} \mid \tau) = \frac{1}{n}\sum_{i=1}^{n} E(X_i \mid \tau) = \tau$. A few people erroneously assumed that the distribution of $\overline{X}$ was the same as that of each X_i (i.e. exponential) and argued that the estimator was unbiased as this distribution had expectation τ. $\overline{X}$ and X_i have the same expectation but not the same distribution. For example, the variance of $\overline{X}$ is $1/n$th that of X_i.