

Additional Handwritten Notes for Lecture Two.

$$X \sim \text{Bin}(n, \theta) \quad f_X(x|\theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

$$Y \sim \text{NBin}(r, \theta) \quad f_Y(y|\theta) = \binom{y-1}{r-1} \theta^r (1-\theta)^{y-r}$$

Observe  $x=r$  and  $y=r$ : two experiments see the same number of successes in the same number of trials

$$\begin{aligned} f_X(x|\theta) &= c' \theta^x (1-\theta)^{n-x} \\ &= c f_Y(y|\theta) \end{aligned}$$

SLP implies make the same inference about  $\theta$  in both cases.

Birnbaum's Theorem.

$$(WIP \cap WCP) \leftrightarrow \text{SLP}$$

Proof.

Both  $\text{SLP} \rightarrow \text{WIP}$  and  $\text{SLP} \rightarrow \text{WCP}$  straightforward. We'll show is that  $(\text{WIP} \cap \text{WCP}) \rightarrow \text{SLP}$ .

Let  $\xi_1$  and  $\xi_2$  be two experiments with

$$f_{\xi_1}(x_1|\theta) = c(x_1, x_2) f_{\xi_2}(x_2|\theta)$$

for  $c = c(x_1, x_2) > 0$ .  $c$  is known as  $x_1, x_2$  have been observed.

Consider the mixture experiment when I perform  $\xi_1$  with probability  $p_1 = \frac{1}{c+1}$

and  $\xi_2$  with probability  $p_2 = \frac{c}{c+1}$

Then:

$$f^*((1, a_1) | \mathcal{Q}) = \frac{1}{c+1} f_{X_1}(a_1 | \mathcal{Q})$$

probability of choosing  $\xi_1$

probability of choosing  $\xi_2$

$$= \frac{c}{c+1} f_{X_2}(a_2 | \mathcal{Q}) \quad (\text{as } f_{X_1}(a_1 | \mathcal{Q}) = c f_{X_2}(a_2 | \mathcal{Q}))$$

$$= f^*((2, a_2) | \mathcal{Q})$$

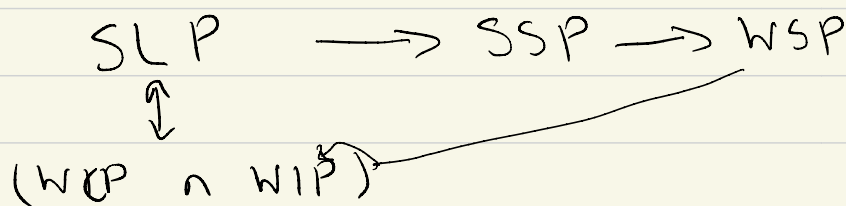
Involves the same experiment  $\xi^*$

By the WIP,  $E_V(\xi^*, (1, a_1)) = E_V(\xi^*, (2, a_2))$

" WCP

$E_V(\xi_1, a_1) \qquad E_V(\xi_2, a_2)$

Putting them together gives  $E_V(\xi_1, a_1) = E_V(\xi_2, a_2)$  which is the SLP.



Example stopping rule

Sequence  $A_i \subset \mathcal{X}_1 \times \dots \times \mathcal{X}_i$  and

$$p_j(a_1, \dots, a_j) = \begin{cases} 0 & \text{if } (a_1, \dots, a_j) \in A_j \text{ STOP} \\ 1 & \text{if } (a_1, \dots, a_j) \notin A_j \text{ CONTINUE} \end{cases}$$