Additional handwritten notes to accompany the Lecture Three Slides.

$$\begin{tikzcd}
 & & \text{SRP} & & \\
\text{SCP} \arrow[dr] & & \text{SLP} \arrow[u] \arrow[ll] \arrow[r] \arrow[dl] \arrow[dr] & \text{SSP} \arrow[r] & \text{WSP} \arrow[dl] \\
 & \text{WCP} & \wedge \arrow[u] & \text{WIP} & \\
 & & \text{BIRNBAUM'S THEOREM} & (\text{DP} \wedge \text{TP}) \arrow[u] &
\end{tikzcd}$$

DP : $\xi = \xi'$ then $Ev(\xi, x) = Ev(\xi', x)$

TP : $g: \mathcal{X} \to \mathcal{Y}$ $\quad Ev(\xi, x) = Ev(\xi^g, g(x))$

WIP : $f_X(x|\theta) = f_X(x'|\theta)$ then $Ev(\xi, x) = Ev(\xi, x')$

WCP : ξ^* mixture, $\quad Ev(\xi^*, (i, x_i)) = Ev(\xi_i, x_i)$

SLP : $f_{X_1}(x_1|\theta) = c(x_1, x_2) f_{X_2}(x_2|\theta)$ then $Ev(\xi_1, x_1) = Ev(\xi_2, x_2)$

SSP : $S = s(x)$ sufficient then $Ev(\xi, x) = Ev(\xi^S, s(x))$

WSP : $s(x) = s(x')$ then $Ev(\xi, x) = Ev(\xi, x')$

SRP : $Ev(\xi^\tau, (x_1, \dots, x_n))$ doesn't depend on SRP.

SCP : Y is ancillary, $\quad Ev(\xi, (x, y)) = Ev(\xi^{X|y}, x)$

Key idea of the Bayesian approach:

as a function of θ

$$\pi(\theta|x) \propto f_X(x|\theta)\,\pi(\theta)$$

Posterior $\propto$ Likelihood $\times$ Prior

Strong likelihood principle is that if, as a function of θ, the likelihoods are proportional then the inference about θ should be the same in each case.

$$\pi_1(\theta \mid x_1) \propto f_{X_1}(x_1 \mid \theta)\,\pi(\theta)$$ (model $\mathcal{E}^{B,1}$)

$$\pi_2(\theta \mid x_2) \propto f_{X_2}(x_2 \mid \theta)\,\pi(\theta)$$ (model $\mathcal{E}^{B,2}$)

SLP: suppose $f_{X_1}(x_1 \mid \theta) \propto f_{X_2}(x_2 \mid \theta)$ as a function of θ.

$$\begin{aligned}\pi_1(\theta \mid x_1) &\propto f_{X_1}(x_1 \mid \theta)\,\pi(\theta)\\ &\propto f_{X_2}(x_2 \mid \theta)\,\pi(\theta)\\ &\propto \pi_2(\theta \mid x_2)\end{aligned}$$

Thus $\pi_1(\theta \mid x_1) = \pi_2(\theta \mid x_2)$. Inferences using $\pi_1(\theta \mid x_1)$ will be identical as those using $\pi_2(\theta \mid x_2)$. Bayesian statistics will satisfy the SLP.