Additional notes to accompany Lecture Six

$$E\{L(\theta,d)\} = \int_{-\infty}^{d} \beta(d-\theta)\pi(\theta)\,d\theta + \int_{d}^{\infty} \alpha(\theta-d)\pi(\theta)\,d\theta$$

$$= \beta d\, P_{(\pi)}(\theta \leq d) - \beta \int_{-\infty}^{d} \theta\,\pi(\theta)\,d\theta + \alpha \int_{d}^{\infty} \theta\,\pi(\theta)\,d\theta - \alpha d\,[1 - P_{(\pi)}(\theta \leq d)]$$

Differentiating with respect to d,

$$\frac{d}{dd} E\{L(\theta,d)\} = \beta P_{(\pi)}(\theta \leq d) + \beta \cancel{d\pi(d)} - \beta \cancel{d\pi(d)} - \alpha \cancel{d\pi(d)}$$

$$- \alpha[1 - P_{(\pi)}(\theta \leq d)] + \alpha \cancel{d\pi(d)}$$

$$= (\alpha+\beta) P_{(\pi)}(\theta \leq d) - \alpha$$

The Bayes rule δ^* satisfies $P_{(\pi)}(\theta \leq \delta^*) = \dfrac{\alpha}{\alpha+\beta}$.

$$L(\theta,d) = (d-\theta)^T Q (d-\theta)$$

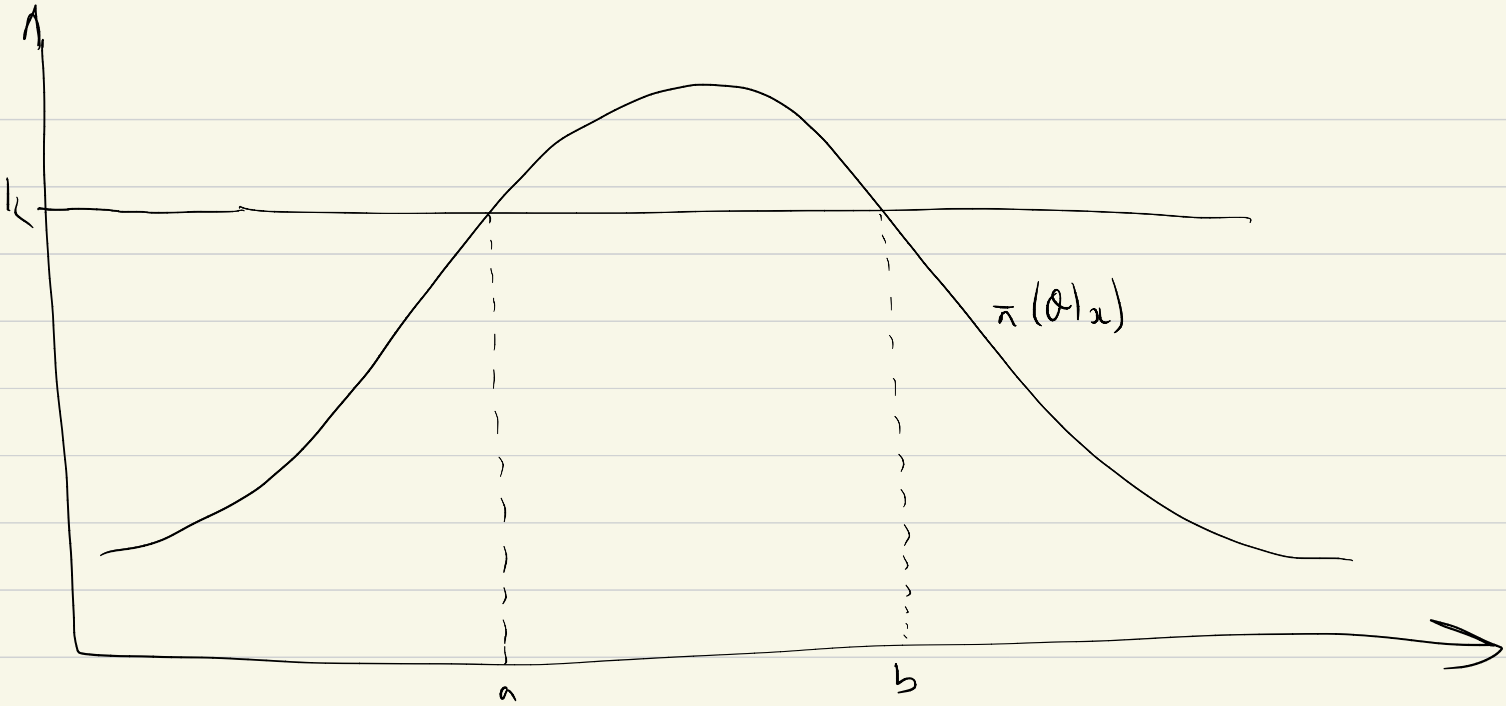
$$= ((d - E_{(\pi)}(\theta)) + (E_{(\pi)}(\theta) - \theta))^T Q ((d - E_{(\pi)}(\theta)) + (E_{(\pi)}(\theta) - \theta))$$

so that

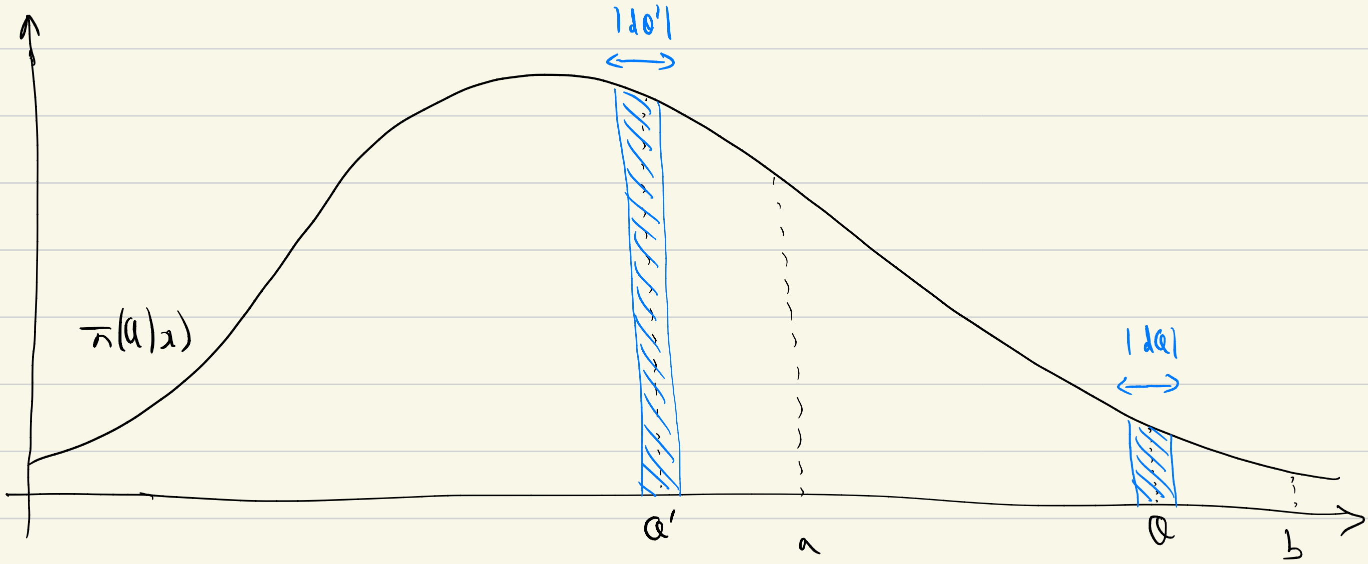
$$E\{L(\theta,d)\} = \overbrace{(d - E_{(\pi)}(\theta))^T Q (d - E_{(\pi)}(\theta))}^{\text{only this term depends upon } d}$$

$$+ \underbrace{E[(E_{(\pi)}(\theta) - \theta)^T Q (E_{(\pi)}(\theta) - \theta)]}_{\text{fixed for all } d}$$

As $(d - E_{(\pi)}(\theta))^T Q (d - E_{(\pi)}(\theta)) \geq 0$ so $d^* = E_{(\pi)}(\theta)$ which does not depend upon Q.



Interval (a,b) is a level set. (HPD region)



Region (a,b) is not a level set $\therefore$ there is a $\theta \in d$ $\quad d = (a,b)$ such that

$\pi(\theta|x) < \pi(\theta'|x)$ for some $\theta' \notin d$

$d' = ((a,b) \setminus |d\theta|) \cup |d\theta'|$ then d has the same volume as d'

but $\mathbb{P}(\theta \in d' | X) > \mathbb{P}(\theta \in d | X)$