Notes to accompany Lecture Seven.

C is a nesting family $\alpha < \alpha'$ then $C(x;\alpha') \subset C(x;\alpha)$

As $\alpha < \alpha'$ then $1-\alpha' < 1-\alpha$ so the coverage of $C(x;\alpha')$ is lower than that of $C(x;\alpha)$

i.e. To increase the coverage, we increase the width.

$$[\longleftarrow C(x;\alpha') \longrightarrow]$$
$$[\longleftarrow\!\!-\!\!-\!\!- C(x;\alpha) -\!\!-\!\!-\!\!\longrightarrow]$$

A PIVOT for the model $\{=\{X, \Theta, f_X(x|\theta)\}$ is a random variable $Q(X,\theta)$ for which the distribution of Q does not depend on θ.

$$\mathbb{P}(\underbrace{a \le Q(X,\theta) \le b}_{\text{Here won't depend on } \theta \text{ if } Q(X,\theta) \text{ is a pivot}} | \theta) \ge 1-\alpha$$

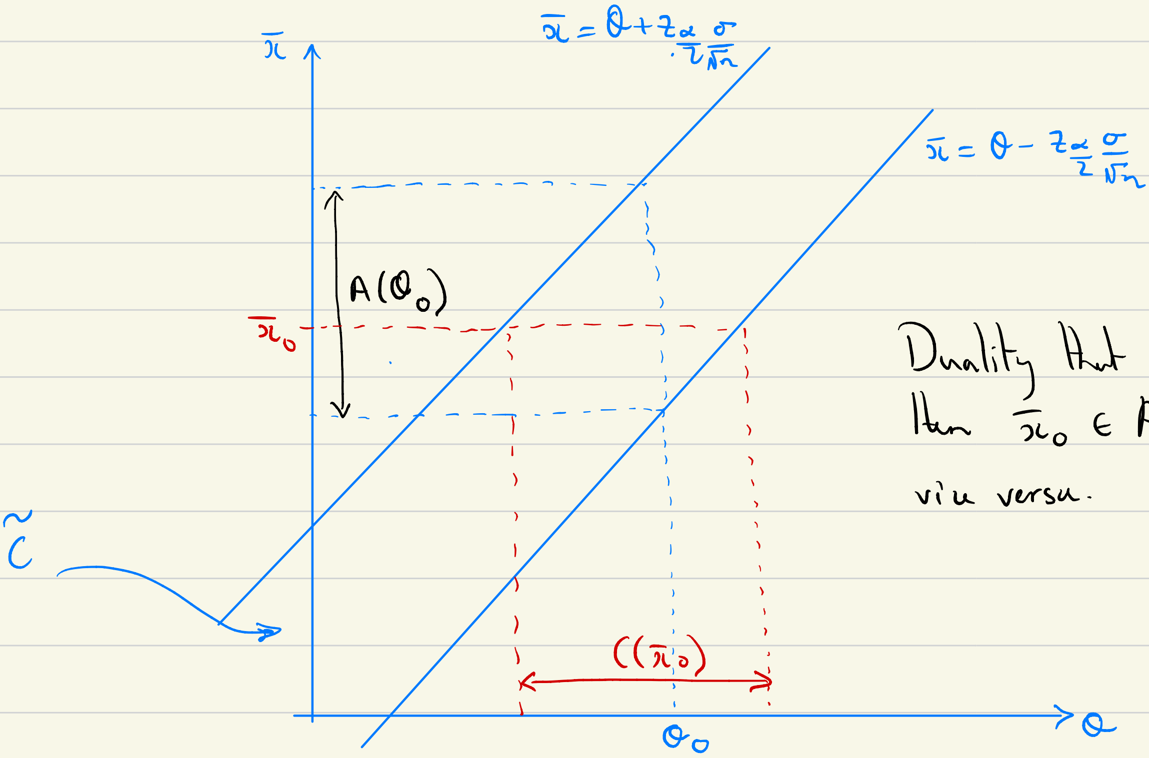
$C(x) = \{\theta : a \le Q(x,\theta) \le b\}$. Idea of PIVOTAL INFERENCE.

$X_1, \dots, X_n$ iid $N(\theta, \sigma^2)$

$Q(X,\theta) = \dfrac{\bar{X}-\theta}{\sigma/\sqrt{n}}$ is a pivot for $X = (X_1, \dots, X_n)$

$$\mathbb{P}\Bigg(\underbrace{-z_{\frac{\alpha}{2}} \le \frac{\bar{X}-\theta}{\sigma/\sqrt{n}} \le z_{\frac{\alpha}{2}}}_{\text{Invert this to express as}} \Bigg| \theta\Bigg) = 1-\alpha$$

$$\mathbb{P}\left(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \le \theta \le \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \,\Big|\, \theta\right) = 1-\alpha$$



Duality that if $\theta_0 \in C(\bar{x}_0)$ then $\bar{x}_0 \in A(\theta_0)$ and via versa.