Notes to accompany Lecture Eight.

$$C(x) = \{ \theta : f_X(x|\theta) > g(x) \}$$

$C(x)$ is constructed as a realisation of the random interval $C(X)$. Thus, in $C(x)$, $f_X(x|\theta) = L_X(\theta; x)$, a function of θ, and $g(x)$ is a constant.

In the linear model, $\hat{\theta}(Y) \sim N_p(\theta, \sigma^2 (X^T X)^{-1})$

$$\hat{\theta}(Y) - \theta \sim N_p(0, \sigma^2 (X^T X)^{-1})$$

$$X(\hat{\theta}(Y) - \theta) \sim N_p(0, \sigma^2 I_p) \quad (*)$$

$$[\; X(X^T X)^{-1} X^T = X X^{-1} (X^T)^{-1} X^T = I_p \;] \quad (*)$$

(*) Post lecture note: (*) is incorrect. It should be $\frac{1}{\sigma}(X^T X)^{1/2}(\hat{\theta}(Y) - \theta) \sim N_p(0, I_p)$

since $\mathrm{Var}\left(\frac{1}{\sigma}(X^T X)^{1/2}(\hat{\theta}(Y) - \theta)\right) = \frac{1}{\sigma^2}(X^T X)^{1/2} \sigma^2 (X^T X)^{-1} (X^T X)^{1/2}$

$$= I_p.$$

Thus, $-2 \log \lambda(Y) = Z^T Z$ where $Z = \frac{1}{\sigma}(X^T X)^{1/2}(\hat{\theta} - \theta)$ and $Z \sim N_p(0, I_p)$.