‘Random Geometric Graphs’
by Mathew Penrose, Oxford University Press 2003:
Errata and Corrections

I thank Sven Ebert, Andy Parrish, Tom Rosoman and Andrew Wade, who between them pointed out many of these errors to me.

Page 19, 14 lines from the end (last displayed line): the $\times$ should be $=$.

Page 34. In the 4th line of the statement of Theorem 2.10, the $D_{n,n}$ should be D_{n,k_n}.

Page 34, last line but 1. In this display the first $M_{n,j}$ shuld be $M'_{n,j}$

Page 36: in Lemma 2.11, λ should be in $(0,\infty)$ rather than $(0,\infty]$.

Page 36, last line but 1. Change W_i to W_1.

Page 37, equation (2.40). The subscript under the sup should be $n - n^\gamma \leq m \leq n + n^\gamma$, not $n - n^\gamma \leq m \leq m' \leq n + n^\gamma$.

Page 42, last line of proof of Lemma 2.14, should perhaps change lim inf to $\liminf_{n\to\infty}$.

Page 53: in Equation (3.12), change the second $E\xi_{\mathbf{i},n}$ to $E\xi_{\mathbf{j},n}$.

Page 76: in the line before Equation (4.4), change r_n to k_n.

Page 177: in the second line of Section 9.1, change $\pi(y) = 1$ to $\pi(1) = y$.

Page 179: in the statement of Corollary 9.4, change $B_{\mathbb{Z}}(m)$ to $B_{\mathbb{Z}}(m_1, \ldots, m_d)$. Also make the same change twice in the proof.

Page 182: in Equation (9.5), the second A should be A^*.

Page 183, 8 lines from the bottom. It is asserted that $B_{\mathbb{Z}}(n) \setminus F_i$ is connected in the lattice, but this is not always true if Λ is not connected. Fixing this error in Lemma 9.10 requires significant extra work which is deferred to later in this document.

Page 184: at the start of the proof of Lemma 9.11, insert the extra sentence: ‘Assume without loss of generality that $P[Z \geq 0] = 1$.’

Page 185, proof of Theorem 9.8: in the tenth line of this proof, just before the word ‘whenever’ it says $n^d/3 < \xi_J \leq (1-\varepsilon)n^d/3$ which should be $n^d/3 < \xi_J \leq (1-\varepsilon)n^d$.

Page 194, just below formula (10.1) it says $L_2(G(n,\lambda/n)) \xrightarrow{P} 0$ but it should say $n^{-1}L_2(G(n,\lambda/n)) \xrightarrow{P} 0$.

Page 201: in Lemma 10.4, change $P[LR_a \to 1]$ to $P[LR_a] \to 1$ and $P[SLR_a \to 1]$ to $P[SLR_a] \to 1$.

Page 202, line 7 from the bottom (3rd line of proof of Lemma 10.5). The $\leq$ should be $\geq$ both times.

Page 203, 4th line from end of proof of Lemma 10.5 (line starting 'by definition'). The $\leq$ should be $\geq$ both times.

Page 204, line 3 from the bottom. $S(K, a)$ should be $S(K_1, a)$.

Page 205, line 4. 'There is no component' should be 'there is a component'.

Page 205, line 10 from the bottom (4th line of Remark): $g(x)$ should be $g_1(x)$.

Page 206, eqn (10.26). In the denominator, n should be s.

Page 206, eqn (10.27). In the denominator, $\emptyset$ should be A the first time and B the second time.

Page 216, line above (10.53). It should say $k(t) \in (c_0 t^d, 2c_0 t^d)$, instead of $k(t) \in (c_0 t, 2c_0 t)$.

Page 216, equation (10.53). Change the last t to t^d.

Page 216, line before (10.54). Change $2(\alpha^{-1} \log s)^{d/(d-1)}$ to $2(2c_0)^{1-1/d}(\alpha^{-1} \log s)^{1/(d-1)}$.

Page 216, equation (10.54). Change $(\log s)^{d/(d-1)}$ to $(\log s)^{1/(d-1)}$.

Page 216, last line. Change $k((2c_0)^{-1}(\alpha^{-1} \log s)^{d/(d-1)})$ to $k((2c_0)^{-1/d}(\alpha^{-1} \log s)^{1/(d-1)})$.

Page 217, line 9. Change $k((2c_0)^{-1}(\alpha^{-1} \log s)^{d/(d-1)})$ to $k((2c_0)^{-1/d}(\alpha^{-1} \log s)^{1/(d-1)})$.

Page 225, Line 3 (statement of Theorem 10.22). It says 'Suppose $d > 2$'. That should be 'Suppose $d \geq 2$'

Page 226, line 8 from the bottom: 'number of points' should be 'set of points'.

Page 227, line 7 from the bottom: 'uniformly in B' should be 'uniformly in A'.

Page 229, line 3 from the bottom: The subscript under $\sum$ should be $x \in B'_n$ not $x \in B_n$.

Page 329, Column 2, Line 17. Index item 'Palm theory, Palm point processes' also features on pages 20 and 191.

Correction to Lemma 9.10

As mentioned above, there is a mistake on page 183 which can be repaired as follows.

Let $\delta_2 := (2d)^{-d/(d-1)}\delta_1/2$. Then the statement of Lemma 9.10 is valid with δ_1 replaced by δ_2. This should not affect the applicability of this lemma later on.

The printed proof is valid in the special case where Λ is connected. In the general case list the components of Λ as $V_1, V_2, \ldots, V_R$.

First suppose $|V_r| > n^d/2$ for some r, say for $r = 1$. Let the $*$-connected components of $\partial^+_{B(n)} V_1$ be denoted $D_1, \ldots, D_J$. Then by applying the special case of the result already mentioned [taking Λ to be V_1 and Λ' to be $B(n) \setminus (V_1 \cup \partial^+_{B(n)} V_1)$] we have that

$$\sum_{j=1}^{J} |D_j|^{d/(d-1)} \geq \delta_1 (n^d - |V_1|)$$

and this implies the result for this case.

Now suppose instead that $|V_r| \leq n^d/2$ for all r. For each r let $W_{r,1}, \ldots, W_{r,S(r)}$ be the components of $B(n) \setminus V_r$. We assert that for all r we have

$$\sum_{s=1}^{S(r)} |\partial^-_{B(n)} W_{r,s}|^{d/(d-1)} \geq \delta_1 |V_r|. \tag{1}$$

To see this, fix r and first suppose that $|W_{r,s}| > n^d/2$ for some s, say for $s = 1$. Then by Lemma 9.9 (taking A to be the complement of $W_{r,1}$) we have

$$|\partial^-_{B(n)} W_{r,1}|^{d/(d-1)} \geq \delta_1 (n^d - |W_{r,1}|) \geq \delta_1 |V_r|$$

which gives us (1) for this case.

Now suppose instead that for all $s \in \{1, \ldots, S(r)\}$ we have $|W_{r,s}| \leq n^d/2$. Then by Lemma 9.9 (taking A to be $W_{r,s}$) we have $|\partial^-_{B(n)} W_{r,s}|^{d/(d-1)} \geq \delta_1 |W_{r,s}|$ and therefore, summing over s, we have

$$\sum_{s=1}^{S(r)} |\partial^-_{B(n)} W_{r,s}|^{d/(d-1)} \geq \delta_1 (n^d - |V_r|) \geq \delta_1 |V_r|$$

where the last inequality is from our assumption on $|V_r|$. Thus we have established (1) for this case too.

For each (r, s), the set $\partial^-_{B(n)} W_{r,s}$ is $*$-connected (because both $W_{r,s}$ and its complement are connected), and is contained in $\partial^+_{B(n)} \Lambda$ so is disjoint from $\Lambda \cup \Lambda'$. Therefore $\partial^-_{B(n)} W_{r,s}$ is contained in one of the C_i, for each (r, s).

If $x \in \partial^-_{B(n)} W_{r,s}$ then x lies adjacent to y for some $y \in V_r$, which is the case for at most $2d$ values of r. Also for any $x \in B(n)$ and any r,

x lies in at most one of the sets $\partial^-_{B(n)} W_{r,1}, \ldots, \partial^-_{B(n)} W_{r,S(r)}$ (because these sets are disjoint). Therefore, each $x \in B(n)$ lies in at most $2d$ of the sets $W_{r,s}, 1 \leq r \leq R, 1 \leq s \leq S(r)$. Hence, for each i,

$$|C_i|^{d/(d-1)} \geq \left(\frac{1}{2d} \sum_{\{(r,s):\partial^-_{B(n)} W_{r,s} \subseteq C_i\}} |\partial^-_{B(n)} W_{r,s}| \right)^{d/(d-1)}$$

$$\geq \left(\frac{1}{2d} \right)^{d/(d-1)} \sum_{\{(r,s):\partial^-_{B(n)} W_{r,s} \subseteq C_i\}} |\partial^-_{B(n)} W_{r,s}|^{d/(d-1)}$$

where for the last line we have used Minkowski's inequality as in the printed proof. Hence, by (1),

$$\sum_i |C_i|^{d/(d-1)} \geq (2d)^{-d/(d-1)} \sum_{r=1}^{R} \sum_{s=1}^{S(r)} |\partial^-_{B(n)} W_{r,s}|^{d/(d-1)}$$

$$\geq (2d)^{-d/(d-1)} \delta_1 \sum_{r=1}^{R} |V_r| = 2\delta_2 |\Lambda| \geq \delta_2 (n^d - |\Lambda|)$$

where the last inequality comes from our assumption that $|\Lambda| > n^d/3$. This gives us the result.

Last updated 5 July 2010
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