OpenMath:

Symbols, CDs and Signatures

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An OpenMath symbol

1. is defined in a Content Dictionary

   along with other symbols;

2. has Formal Mathematical Properties;

3. has a signature (in STS file);

4. may have other features in other files.

OpenMath is not parsimonious

"$\leq$ is unnecessary":

$$a \leq b \Leftrightarrow (a < b) \vee (a = b).$$

True, but if a and b are large, we might write

$$(\lambda xy.(x < y) \vee (x = y))ab$$

for brevity, and then why not call the λ-expression $\leq$?

OpenMath can/should nod to notation (1)

"$<$ is unnecessary":

$$a < b \Leftrightarrow b > a.$$

True, but $<$ is sufficiently entrenched that, much as we might wish it had never been invented, it has been!

OpenMath can/must nod to notation (2)

- Everyone teaches that $T(n) = O(n^2)$ is an abuse of notation, and then (with one honourable exception), abuses it.

- This is certainly *not* `<OMS name="eq" cd="relation1'`

- **Semantically**, it certainly *is* `<OMS name="in"` `cd="set1"/>`.

So `<OMS name="Landauin" cd="asymp?"/>`, whose semantics are those of `<OMS name="in" cd="set1"/>`

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CDs: Large or Small

Large All the world is one CD — the MathML 1 mistake.

Small Every symbol in its own CD. Possible, and there probably will be CDs with only one symbol, but JHD hopes that these would be transitory: "we need more symbols about aardvark-stuffing than are in `aardvark1`, but so far I can only think of `chilli`".

Right-sized is in the eye of the beholder.

Which CD?

`<OMS name="Landauin" cd="asymp?"/>`

asymp1− An (upwards-compatible) change.

asymp1+ It is hard to use `asymp1` without it.

asymp2+ "Natural evolution".

I think `asymp1` is the right solution.

Fitness for purpose

`poly` I am interested in polynomials.

`polyr` I am interested in polynomials in $\mathbf{Z}[x_1][x_2]\dots$.

`polyd` I am interested in polynomials in $\mathbf{Z}[x_1, x_2 \dots]$.

⋮ Other views.

Missing facts (in `relation1`)

eq Currently transitive, not reflexive, symmetric.

- A bug, which needs fixing.

- Sufficiently fundamental that a new major version is called for.

lt etc. Currently transitive, not specified as a total order.

- Is this a bug, which needs fixing?

Yes of course it is!

No: You never said it was, and I use it all through my book on posets.

Plan A new CD `relation5` with $a <_5 b \Rightarrow a <_1 b$.

This CD has total order FMPs. Note that it is impossible to write an FMP for $<$ that says $<$ is a total order without involving other symbols.

If it's important to you that the order is total, use `relation5`.

Signatures — STS

- Very limited goals.

- Different files from CD, but linked by names.

- Allows some machine checking.

  (Does anyone do this?)

- Can inform a human being — which order
  are the arguments to `<OMS name="E" cd="expint"/>`?

  A natural number followed by a function.

- Users are free to ignore STS.

Is there a moral here?

Goals for presentation methods

- Very limited goals.

- Different files from CD, but linked by names.

- Translate into (enhanced) MathML-P.

- How to go to LaTeX etc.?

* After the meeting, PL said this was trivial for his system.