AISC Meets Natural Typography

James H. Davenport
Department of Computer Science
University of Bath
Bath BA2 7AY England
J.H.Davenport@bath.ac.uk

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but some abuse is more harmful than others.

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Inverse functions Is arcsin the (a?, which??) single-valued inverse and Arcsin the multi-valued (Anglo-Saxon), or the converse (French)?

i **or** j In practice, this causes little confusion for experts, some for students.

metric tensor Is the metric tensor for flat Minkowski space $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ or its negative $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$? Is the temporal variable the last, rather than the first, coordinate, giving $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, or its negative?

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`\left(\frac{-1}{p}\right)` is, alas, how professional typesetters encode it.

"this has the usual mathematical meaning"
(1)

Mathematics $a_1 \cup a_2 \cup a_3$

LaTeX `a_1 \cup a_2 \cup a_3`

OpenMath `<OMS name="union" cd="set1"/>`

MathML `<apply> <union/> <i>`a_1`</i>...</apply>`

"this has the usual mathematical meaning"
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Mathematics $\bigcup\{a_1, a_2, a_3\}$

LaTeX `\bigcup \{a_1,a_2,a_3\}`

OpenMath `<OMS name="big_union" cd="set3"/>`

or `<OMS name="apply_to_list" cd="fns2"/>`

MathML `<apply> <union/> <bvar>i</bvar> <domain ...> <set> <i>` a_1 `</i>...</set>`

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Mathematics $\bigcup_{i=1}^{3} a_i$

LaTeX `\bigcup_{i=1}^3 a_i`

OpenMath `big_union` on `make_list`

MathML `<apply> <union/> <bvar>i</bvar> <lowlimit`

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since $p, q, \in \{s, c, n, d\}$. But when $q = s$

$$\mathsf{Pq}(u) = \int_0^u \left(\mathsf{pq}^2(t) - \frac{1}{t^2}\right)\mathrm{d}t - \frac{1}{u}.$$

Pq and friends (2)

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(except that here there is no distinctness assumption, but pp is to be taken as the constant function 1).

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(Function) Application $\sin x$ otherwise $\mathrm{sin}(x)$. $\sin(x+y)$ is different from $2(x+y)$, and $f(x+y)$ is ?? This is encoded as `&ApplyFunction;`.

Addition $4\frac{1}{2}$ could otherwise be rendered as $4+\frac{1}{2}$. This is (now) encoded as `&InvisiblePlus;`.

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Concatenation m_{12} could be rendered as $m_{1,2}$. This is encoded as `&InvisibleComma;`. Even without this, MathML is less ambiguous than ordinary notation: m_{12} might equally be the twelth item of a vector, but MathML would distinguish the following.

```
<msub>
  <mi> m </mi>
  <mrow>
    <mn> 1 </mn>
    <mn> 2 </mn>
  </mrow>
</msub>
```

```
<msub>
  <mi> m </mi>
  <mrow>
    <mn> 12 </mn>
  </mrow>
</msub>
```

(of course, the `<mrow>` is redundant in this case).

An awful example.

Then the functor $T \mapsto \{$generically smooth T-morphisms $T \times_S \mathcal{C}' \to \mathcal{T} \times_{\mathcal{S}} \mathcal{C}\}$ from ((S-schemes)) to ((sets)) is

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The meanings of $\pm$

$$\operatorname{Arcsinh} z_1 \pm \operatorname{Arcsinh} z_2 = \operatorname{Arcsinh}\left(z_1\sqrt{1-z_2^2} \pm z_2\sqrt{1}\right.$$

means

$$\begin{aligned}\operatorname{Arcsinh} z_1 + \operatorname{Arcsinh} z_2 &\subset \operatorname{Arcsinh}\left(z_1\sqrt{1-z_2^2} + z_2\sqrt{1}\right.\\ &\cup \operatorname{Arcsinh}\left(z_1\sqrt{1-z_2^2} - z_2\sqrt{1}\right.\end{aligned}$$

and the fact that the same equation holds for $\operatorname{Arcsinh} z_1 - \operatorname{Arcsinh} z_2$.

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  Slightly more semantic LaTeX

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$(a|b)$

The DML trichotomy

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3. born-digital, i.e. not just the pixels, but the whole workflow.

4. born-intelligent-digital, with semantics recoverable from the markup.