

March 2002 Mock Exam

Hungarian Mathematical Olympiad Round 3

A four hour examination

1. The escribed circle to the side AB of $\triangle ABC$ touches the lines AB , AC (produced) at P , Q respectively. The escribed circle to the side BC touches the lines AC , AB at U , X respectively. Prove that the intersection of PQ and UX is as far from AB as from BC .
2. For which natural numbers n , if any, is there an n -gon containing exactly $n^2 - 30n + 236$ acute angles?
3. Let $n > 1$ be a fixed integer. Find x_1, \dots, x_n real numbers so that

$$\sum_i x_i = 2(n-1), \quad \sum (x_i - 1)^2 = n$$

and x_n is maximized subject to these conditions.