7. (a) If K is a subgroup of the finite group G, then $|K| \mid |G|$. [2]

(b) $\theta : G \to H$ is a group hom^m [2]

$\iff$ $\theta(xy) = \theta(x)\,\theta(y)$ $\forall x, y \in G$

(c). If $x \neq y$, then $xy^{-1} \neq 1$.

Let $z = xy^{-1}$, then

$\theta(z) = \theta(xy^{-1}) = \theta(x)\,\theta(y^{-1})$ (proved in course)

$= \theta(x)\,\theta(y)^{-1}$ [3]

$= 1_H$.

(d) $\exists\ \lambda, \mu \in \mathbb{Z}$ s.t. $\lambda|G| + \mu|H| = 1$ since $|G|$ & $|H|$ are coprime.

Now $\theta(g) = \theta(g^1) = \theta(g^{\lambda|G| + \mu|H|})$

$= \theta(g^\lambda)^{|G|}\,\theta(g^\mu)^{|H|}$

$= \theta(g^{|G|})^\lambda\,\theta(g^\mu)^{|H|}$ [3]

$= \theta(1_G)^\lambda\,\theta(g^\mu)^{|H|}$

$= 1_H \cdot 1_H = 1_H$.