

Olympiad Maths Club Sheet 3

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Entertainments

1. Three cyclists cycle round a circuit consisting of three straight line segments AB , BC and CA . The speeds of the first cyclist on these segments are 12, 10 and 15 kph respectively. The speeds of the second cyclist on these segments are 15, 15 and 10 kph respectively. The speeds of the first cyclist on these segments are 10, 20 and 12 kph respectively. Given that the three cyclists finish simultaneously, find angle ABC .
2. Prime numbers p, q and r satisfy
 - (a) $p + q < 111$ and
 - (b) $(p + q)/r = p - q + r$.Find the largest possible value of pqr .
3. You have an unlimited supply of the digit 3, but only one digit 4. You wish to use these digits to create a (base 10) number which is divisible by as many of the numbers from 1 to 9 as possible. What is the smallest number you can make which satisfies this condition?
4. A car journey takes more than one hour. During any period of one hour during the journey, the average speed of the car is 80kph. Is it possible that the average speed for the whole journey is 100kph? *This happens on a private road, so speed limits do not apply.*
5. Is it possible to put the numbers from 1 to 15 into the cells of a 3×5 rectangular table (3 rows, 5 columns) so that either
 - (i) Each column sum is the same and each row sum is the same?
 - (ii) Each row sum is the same as each column sum?
6. A ten digit positive integer is *Bathonian* when each of its digits is 1, 2 or 3 and every pair of consecutive digits differ by 1.
 - (a) How many Bathonian numbers exist?
 - (b) Prove that the sum of all Bathonian numbers is divisible by 1408.