Boolean Algebra Information Sheet

Suppose that A, B and C are sets, and they are all subsets of a set U which we will call the *universal set*. If you decide to work within a universal set, you have the notion of the *complement* of a set in U. The complement of A is denoted A', $\overline{A}$ or A^c depending on which book you are reading, and is defined by

$$A^c = U \setminus A = \{x \mid x \in U, x \notin A\}.$$

Here is a list of laws which these set operations obey. In each case, the law can be proved by using the definition (given in Lecture 1) of the equality of sets.

1. $A \cup U = U$ and $A \cap U = A$ for all sets A. (universal set laws)
2. $A \cup \emptyset = A$ and $A \cap \emptyset = \emptyset$ for all sets A. (empty set laws)
3. $A \cup A = A \cap A = A$ for all sets A. (idempotent laws)
4. $A \cup B = B \cup A$ and $A \cap B = B \cap A$ for all sets A and B. (commutative laws)
5. $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$ for all sets A, B and C. (associative laws)
6. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. (distributive laws)
7. $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ for all sets A and B. (complementation laws)

In fact one take the intersection and union of arbitary collections of subsets of U, even infinite collections. Choose I a set of labels (often we use $I = \mathbb{N}$). Suppose that for each $i \in I$ we have $A_i \subseteq U$. We can define

$$\bigcup_{i \in I} A_i = \{x \mid x \in A_i \text{ for some } i \in I\} \text{ and } \bigcap_{i \in I} A_i = \{x \mid x \in A_i \text{ for all } i \in I\}.$$

The set I may be infinite, but the obvious generalizations of the seven laws stated above all hold.