Article: CJB/248/2012

# The H Conic

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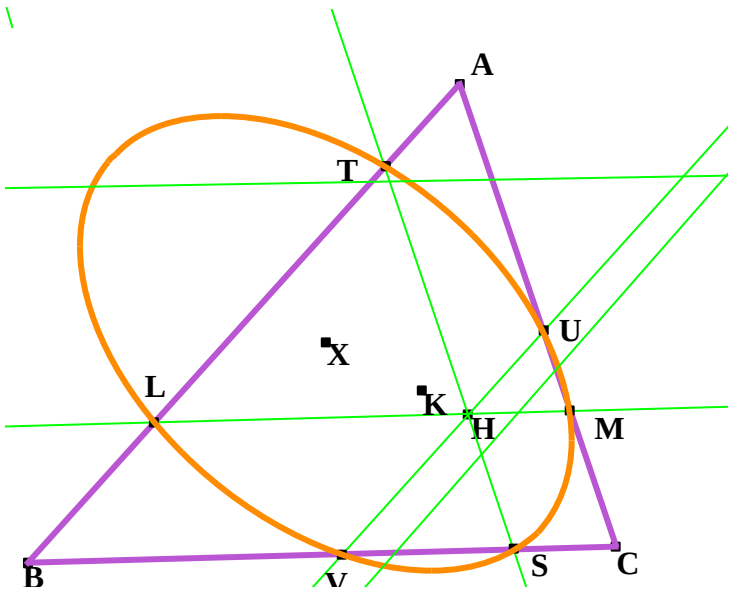
Abstract: Lines are drawn parallel to the sides through the orthocentre H of a triangle ABC meeting the sides in six points. The conic through these six points we term the H conic. We obtain its equation and the co-ordinates of its centre X. It is shown that X, H and K, the symmedian point, are collinear.




**The H Conic**

## 1. The co-ordinates of the six points

Initially we take the co-ordinates of H to be (u, v, w). The equation of the line through H parallel to the x-axis is

$$(v + w)x = u(y + z). \qquad (1.1)$$

This meets the side CA, y = 0, at the point M(u, 0, v + w) and it meets the side AB, z = 0 at the point L(u, v + w, 0).

Similarly the co-ordinates of T are (w + u, v, 0) and those of S are (0, v, w + u). Those of V are (0, u + v, w) and those of U are (u + v, 0, w).

## 2. The equation of the H Conic

The H conic has an equation of the form

$$lx^2 + my^2 + nz^2 + 2fyz + 2gzx + 2hxy = 0. \qquad (2.1)$$

By substituting in the co-ordinates of the six points L, M, S, T, U, V we find

$$l = 2vw(v + w),\ m = 2wu(w + u),\ n = 2uv(u + v), \qquad (2.2)$$

$$f = -u(u^2 + uv + uw + 2vw),\ g = -v(v^2 + vw + vu + 2wu),\ h = -w(w^2 + wu + wv + 2uv). \qquad (2.3)$$

For the point H we have

$$u = 1/(b^2 + c^2 - a^2),\ v = 1/(c^2 + a^2 - b^2),\ w = 1/(a^2 + b^2 - c^2). \qquad (2.4)$$

## 3. The centre X of the H conic

The co-ordinates of the centre X of the conic (2.1) are

$$x = mn - gm - hn - f^2 + fg + hf, \qquad (3.1)$$

$$y = nl - hn - fl - g^2 + gh + gf, \qquad (3.2)$$

$$z = lm - fl - gm - h^2 + hf + hg. \qquad (3.3)$$

Substituting in these equations values of a, b, c and simplifying we find

$$x = (a^4 + a^2(2b^2 + 2c^2) - 3b^4 - 2b^2c^2 - 3c^4)/(b^2 + c^2 - a^2)^2, \qquad (3.4)$$

with y, z following by cyclic change of a, b, c.

It may now be checked that X, H and $K(a^2, b^2, c^2)$ are collinear.

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