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Pascal points

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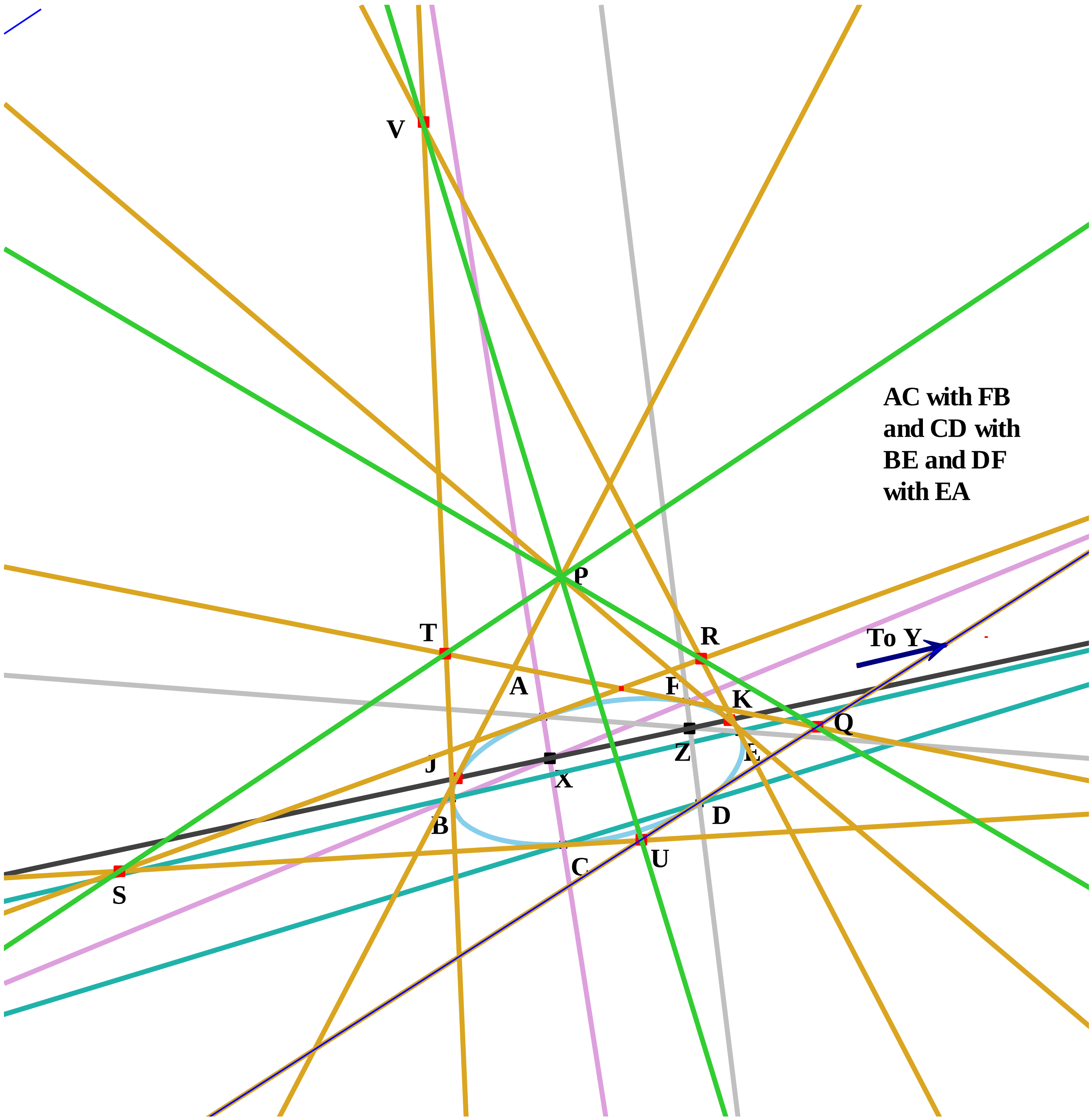
Abstract: An analysis of a Pascal point is given, using the conic $y^2 = zx$ and general points.



The Pascal line XYZ and the Pascal point P

1. Introduction

The conic $y^2 = zx$ is given with general points A, B, C, D, E, F having co-ordinates $A(1, a, a^2)$ etc.

We use the sides AC, FB meeting at X, the sides CD, BE meeting at Y and sides DF, EA meeting at Z to define the Pascal line XYZ.

Tangents at A and C meet at S and those at F and B meet at T. Tangents at C and D meet at U and those at B and E meet at V. Tangents at D and F meet at Q and those at E and A meet at R. Then lines ST, UV, QR are concurrent at P the Pascal pole of XYZ.

The aim of this article is to provide a general analysis.

2. Points A, B, C, D, E, F and the tangents at those points

The conic we have chosen for the general analysis is $y^2 = zx$ and the six points on it A, B, C, D, E, F, have co-ordinates $A(1, a, a^2)$ etc.

The tangent at A has equation

$$2ay = a^2x + x, \qquad (2.1)$$

with similar equations for the other tangents.

Note that the equation of AB is

$$abx + z = (a + b)y. \qquad (2.2)$$

3. The points X, Y, Z and the Pascal line XYZ

X = AC^FB has co-ordinates (x, y, z), where

$$x = a - b + c - f,\ y = ac - bf,\ z = abc + acf - abf - bcf. \qquad (3.1)$$

Y = CD^BE has co-ordinates (x, y, z), where

$$x = c - b + d - e,\ y = cd - be,\ z = bcd + cde - bce - bde. \qquad (3.2)$$

Z = DF^EA has co-ordinates (x, y, z), where

$$x = a - d + e - f,\ y = ae - df,\ z = ade + aef - adf - def. \qquad (3.3)$$

The Pascal line XYZ has equation

$$(abce - abef - acde + acdf + bcdf + bdef)x + (abf - abc - acf + ade - adf + aef + bcd - bce + bcf - bde + cde - def)y + (ac - ae + be - bf - cd + df)z = 0. \qquad (3.4)$$

4. Lines ST, UV, QR and the Pascal point P

The tangents at A and C meet at S with co-ordinates $(1, \frac{1}{2}(a + c), ac)$.
The tangents at F and B meet at T with co-ordinates $(1, \frac{1}{2}(f + b), fb)$.

The equation of ST is

$$(bcf - abc + abf - acf)x + 2(ac - bf)y + (b + f - a - c)z = 0. \quad (4.1)$$

The equation of UV is similarly

$$(bce - bcd + bde - cde)x + 2(cd - be)y + (b + e - c - d)z = 0. \quad (4.2)$$

The equation of QR is likewise

$$(ade - adf + aef - def)x + 2(df - ae)y + (a + e - d - f)z = 0. \quad (4.3)$$

ST, UV, QR concur at the Pascal point P with co-ordinates (x, y, z), where

$$x = ac + be + df - ae - bf - cd, \quad (4.4)$$

$$y = (1/2)(abc - abf + acf + adf - ade - aef + bde - bcd + bce - bcf - cde + def), \quad (4.5)$$

$$z = (abce - abef + acdf - acde + bdef - bcdf). \quad (4.6)$$

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