

# The Second Brocard Triangle

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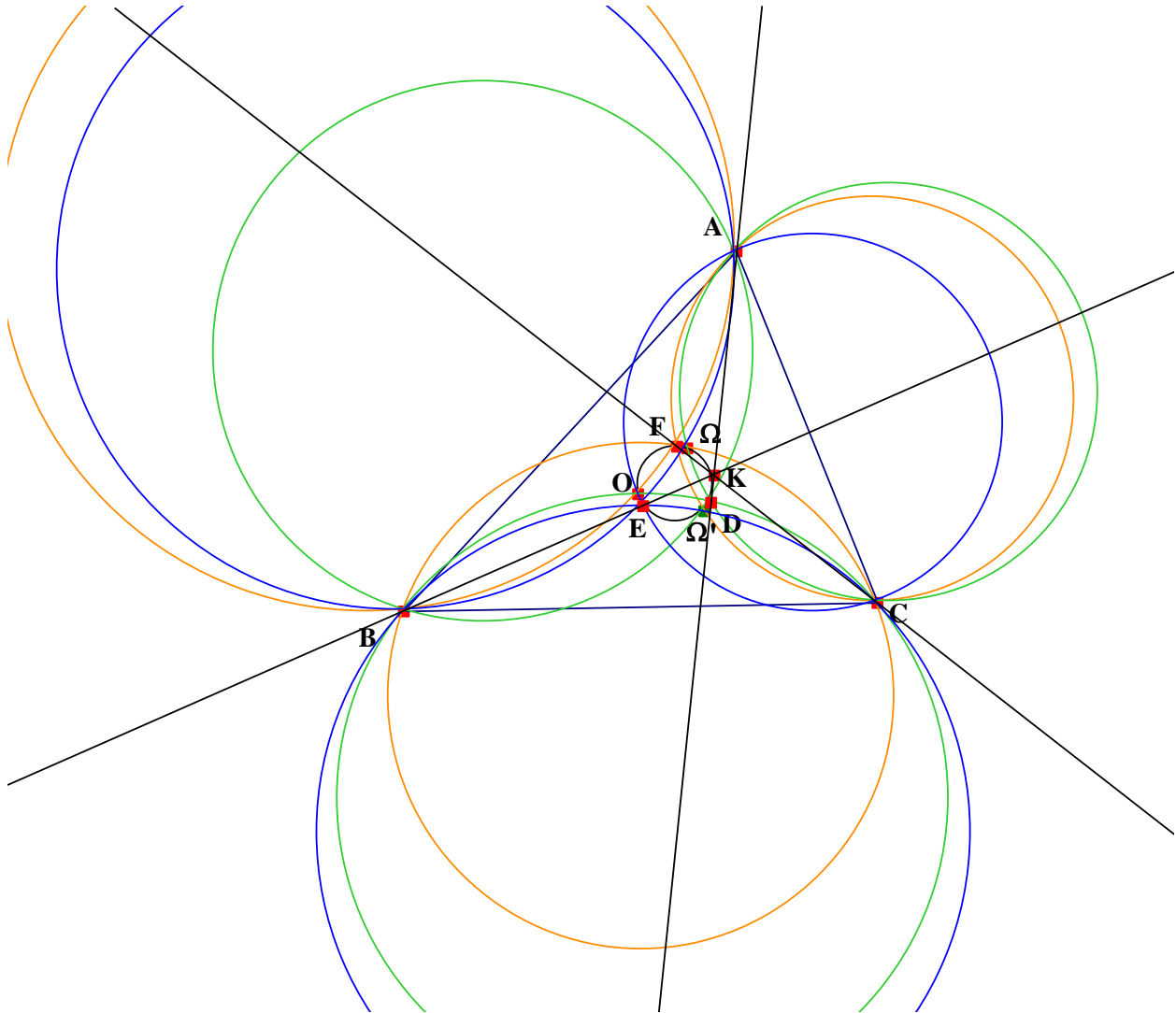


Fig. 1

## Circle Intersections leading to the Second Brocard Triangle

Abstract: Given the Brocard points  $\Omega$ ,  $\Omega'$  in a triangle the point of concurrence of the intersection of the line  $AK$  with the 7-point circle and circles  $AQC$ ,  $BOC$ ,  $AQB$  ( $D$  in the above diagram) and two similar points  $E$ ,  $F$  found by cyclic change of  $A$ ,  $B$ ,  $C$  forms what is known as the Second Brocard Triangle (David Monk – private communication). An analysis is the content of this document.

### 1. Intersections of $AK$ , $BK$ , $CK$ with the 7-point triangle

The equation of the 7-point circle in areal co-ordinates is known to be

$$b^2c^2x^2 + c^2a^2y^2 + a^2b^2z^2 - a^4yz - b^4zx - c^4xy = 0. \quad (1.1)$$

The line AK has equation  $c^2y = b^2z$  and this meets the 7-point circle at a second point D with co-ordinates  $(x, y, z)$ , where

$$x = b^2 + c^2 - a^2, y = b^2, z = c^2. \quad (1.2)$$

Points E, F have co-ordinates which follow from those of D by cyclic change of  $a, b, c$  and  $x, y, z$ .

## 2. The circle BOC contains D

Circles in areal co-ordinates have equations of the form

$$a^2yz + b^2zx + c^2xy + (x + y + z)(ux + vy + wz) = 0. \quad (2.1)$$

Circle BOC has an equation of this form with

$$u = b^2c^2/(a^2 - b^2 - c^2), v = 0, w = 0. \quad (2.2)$$

It may now be checked that point D lies on this circle. Similarly point E lies on circle COA and point F lies on circle AOB.

## 3. The circle AQC contains D

Circle AQC has an equation of the form (2.1) with

$$u = 0, v = -c^2, w = 0. \quad (3.1)$$

It may now be checked that point D lies on this circle. Similarly point E lies on circle BQA and point F lies on circle CQB.

## 4. The circle AQ'B contains D

Circle AQ'B has an equation of the form (2.1) with

$$u = 0, v = 0, w = -b^2. \quad (4.1)$$

It may now be checked that point D lies on this circle. Similarly point E lies on circle BQ'C and point F lies on circle CQ'A.

The triangle DEF is called the Second Brocard Triangle (David Monk – private communication).

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