Article: CJB/2011/178

More Circles centred on the Brocard Axis

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Abstract: Circles through B and C and the Brocard points lead to two circles centred on the Brocard axis and several concurrences.

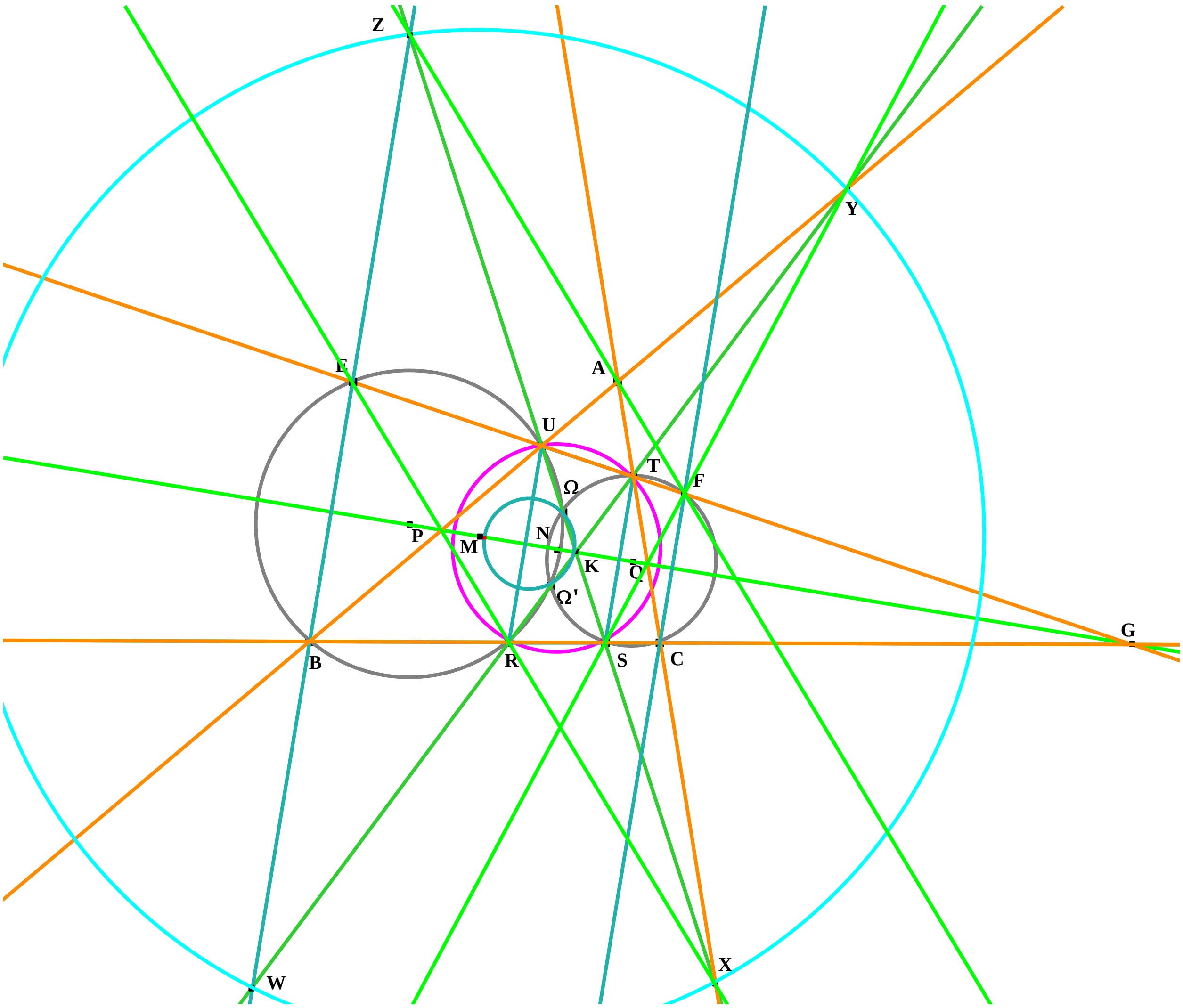

Fig.1
Circles centred on the Brocard axis

Detailed description

What is described now has been verified using areal co-ordinates with ABC as triangle of reference. We use the notation for the Brocard points $\Omega(1/b^2, 1/c^2, 1/a^2)$ and $\Omega'(1/c^2, 1/a^2, 1/b^2)$.

First circles BΩΩ' and CΩΩ' are drawn. BΩΩ' meets BC at R and AB at U. CΩΩ' meets BC at S and CA at T. It then transpires that points R, S, T, U are concyclic. The line TU is now drawn and it meets BΩΩ' at E, CΩΩ' at F and BC at G.

It may now be proved that the four lines BE, RU, ST, CF are parallel.

The diagonals of RSTU are now drawn and it turns out that US passes through Ω and the symmedian point K and that RT passes through Ω' and K.

Points W, X, Y, Z are now obtained with W = BE^RT, X = ER^US^CT, Y = AB^TR^FS and Z = BE^US^AF. It then turns out that W, X. Y. Z are concyclic.

Finally the four circles BΩΩ', CΩΩ', RSTU and WXYZ may be shown to have centres on the Brocard axis OK, where O is the circumcentre of ABC.

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