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The Harmonic Cevian Conic

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Abstract: If D, E, F are the feet of Cevians with D on BC etc. and if L is the harmonic conjugate of C with respect to B and D and U is the harmonic conjugate of B with respect to C and D and if M, N, V, W are similarly defined then a conic passes through the six points L, U, M, V, N, W. There is one internal Cevian point for which the conic is a circle.

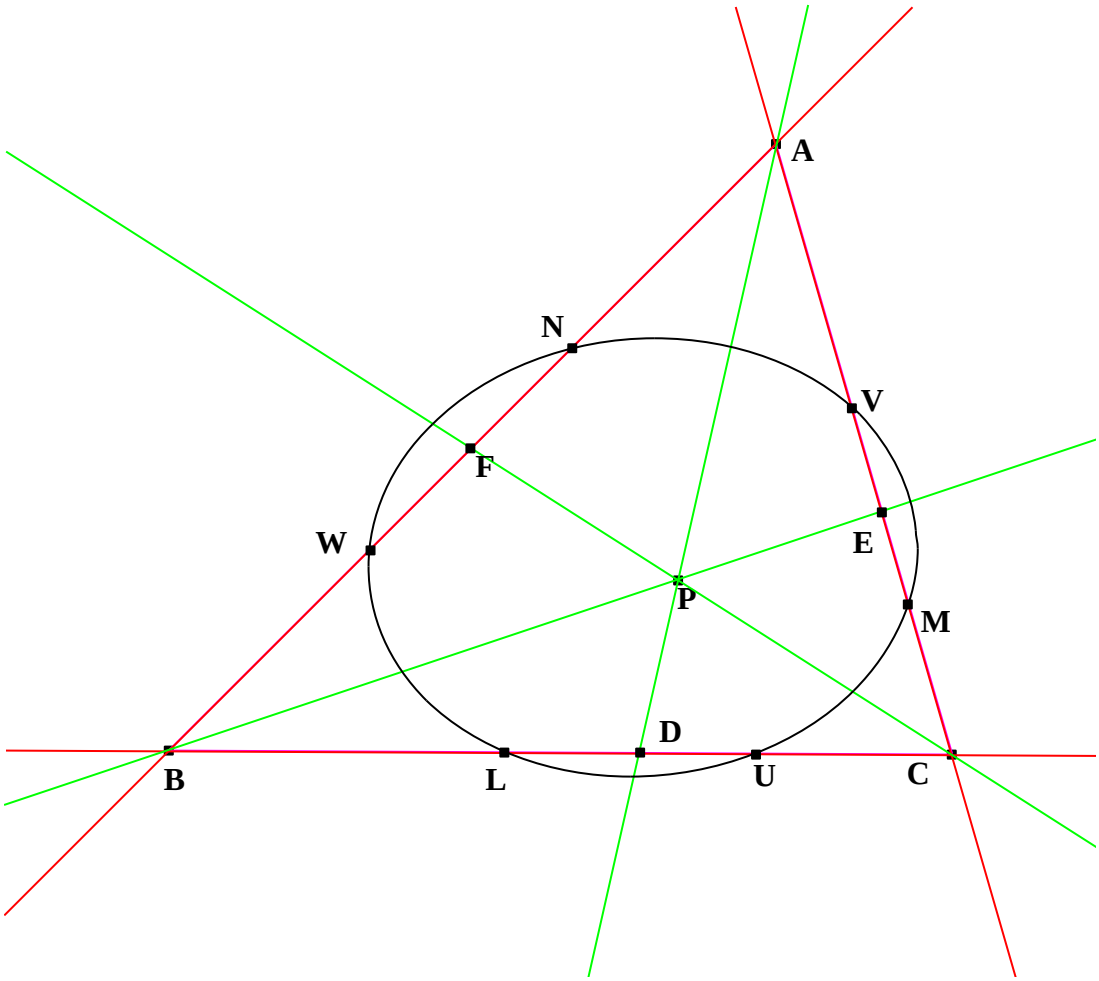


Fig. 1
The Harmonic Cevian Conic

1. Deriving the equation of the Harmonic Cevian Conic

In areal co-ordinates the equation of a conic is of the form

$$px^2 + qy^2 + rz^2 + 2uyz + 2vzx + 2wxy = 0. \qquad (1.1)$$

Here p, q, r, u, v, w are constants to be determined by the co-ordinates of five points lying on the conic.

Suppose P has co-ordinates (f, g, h), then D, E, F have co-ordinates D(0, g, h), E(f, 0, h), F(f, g, 0). U is the harmonic conjugate of B with respect to C and D and therefore has co-ordinates U(0, g, 2h) and L has co-ordinates L(0, 2g, h). Similarly V, W, M, N have co-ordinates V(2f, 0, h), W(f, 2g, 0), M(f, 0, 2h), N(2f, g, 0).

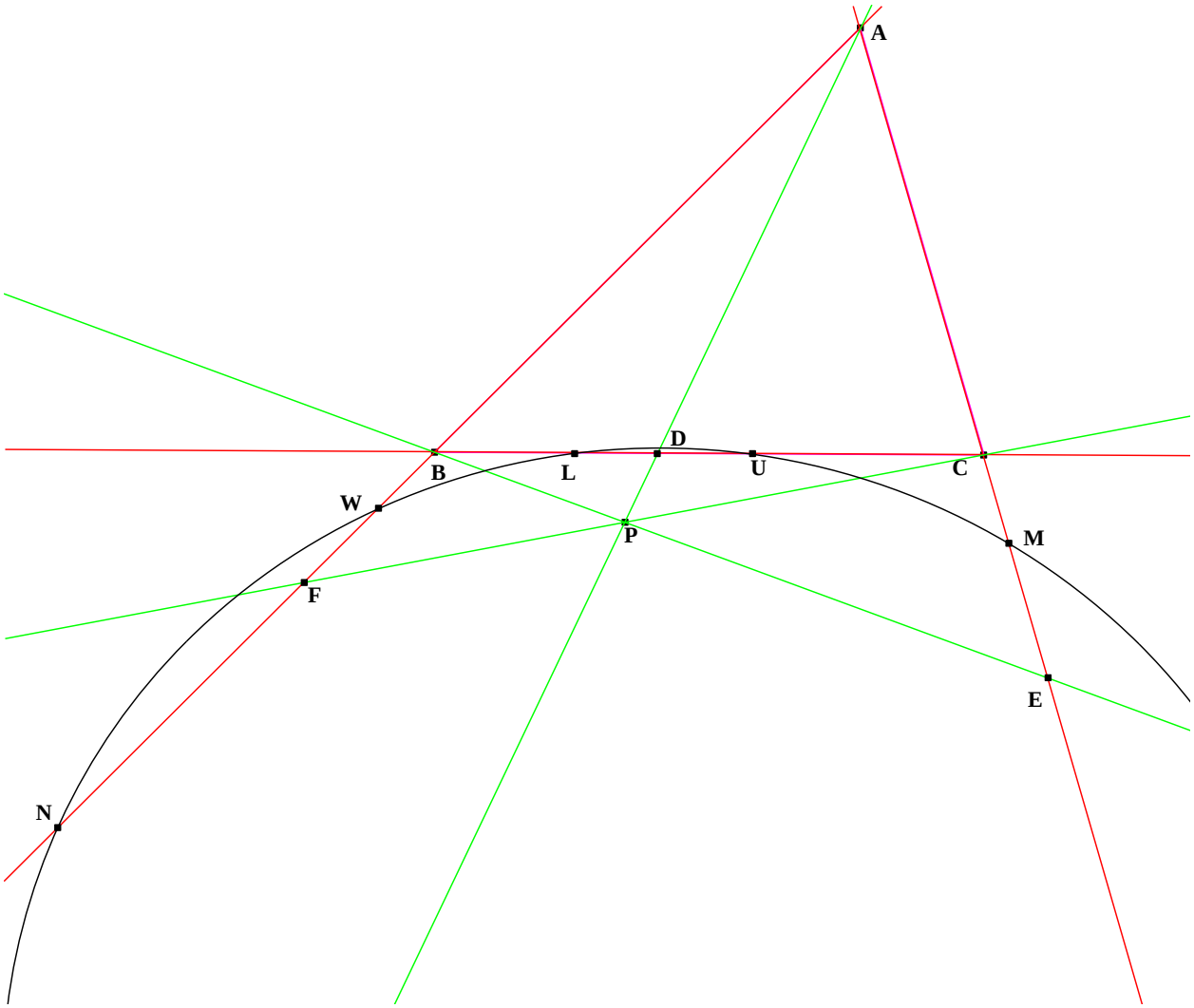
Inserting these co-ordinates into Equation (1.1) and solving we find

$$p = 4g^2h^2,\ q = 4h^2f^2,\ r = 4f^2g^2,\ u = -5f^2gh,\ v = -5fg^2h,\ w = -5fgh^2. \qquad (1.2)$$

This conic is a circle if and only if

$$f^2(4g^2 + 10gh + 4h^2) = a^2,\ g^2(4h^2 + 10hf + 4f^2) = b^2,\ h^2(4f^2 + 10fg + g^2) = c^2. \qquad (1.3)$$

Cabri II plus indicates there are indeed four points P which gives rise to a circle, one internal point and three external points. See Fig. 2.



Fig, 2
One of the Harmonic Cevian Circles

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