University of Bath

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DEPARTMENT OF COMPUTER SCIENCE EXAMINATION

CM20019: Formal Logic and Semantics

dwk, dd January 2003, time

No calculators may be brought in or used.

Full marks will be given for correct answers to THREE questions. If you opt to answer more than the specified number of questions, you should clearly identify which of your answers you wish to have marked. In cases where you have failed to identify the correct number of answers the marker is only obliged to consider the answers in the order they appear up to the number of answers required.

1. (a) Define the notion of a most general unifier for two terms A and B in a term language. [6]

   (b) Either find a most general unifier, or prove that there is no unifier, for $g(g(h(X), Y), Z)$ and $g(Z, g(A, h(B)))$. [7]

   (c) Either find a most general unifier or prove that there is no unifier, for $((X + Y) * Z)$ and $(Z * (X * W))$. [7]

2. (a) Translate the following sentence into some first order language: "The greatest of all detectives played the violin." [8]

   (b) Find clausal form for the result of Part (a). [6]

   (c) Discuss the computational problem of finding clausal form for a formula in a first order language. [6]

3. (a) Give a grammar for the set of (unabbreviated) lambda terms for the pure lambda calculus. [5]

   (b) State the Church-Rosser Theorem. [5]

   (c) First restore brackets, and then find β normal form for $(\lambda pca.pca)(\lambda xy.x)MN$. [6]

   (d) Assume that G does not have a free occurrence of the variable x. Do two β reductions on $(\lambda x.G(xx))(\lambda x.G(xx))$.[4]

4. (a) Let L be a first order language with function symbols h and k, both with arity 2, and written in the usual prefix format, e.g. $h(X, Y)$, $k(X, Y)$, and with predicate symbols p and q, both with arity 2, and written in the usual prefix format, e.g. $p(X, Y)$, $q(X, Y)$. By giving a grammar, or in some other way, explain clearly what are the *terms*, the *atomic formulae* and the *formulae* of L. [6]

   (b) Explain what it means to say that a semantic tableau is satisfiable. [4]

   (c) Assume the One Hop Lemma, that is, that if a semantic tableau T is satisfiable, and $T \Rightarrow T'$ by one application of the semantic tableaux rules, then the semantic tableau T' is also satisfiable. Hence prove correctness for the semantic tableaux system. [10]

5. (a) Write a Prolog program which defines the predicate sublist(L1, L2), which is true if L1 and L2 are lists and every member of L1 is also a member of L2. You may use the standard predicate member(X,Y) which is true if X is a member of list Y. [6]

(b) Use the semantic tableaux method either to prove that the following is logically valid or to give a counterexample:
$((A \rightarrow (B \rightarrow (C \rightarrow D))) \rightarrow ((A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow D)))$ [7]

(c) Use the semantic tableaux method either to prove that the following is logically valid or to give a counterexample:
$(((\exists X)a(X) \rightarrow (\exists X)b(X)) \rightarrow (\exists X)(a(X) \rightarrow b(X)))$ [7]

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